Thermomagnetic convection and open questions in rheology of ferrocolloids

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Outline:

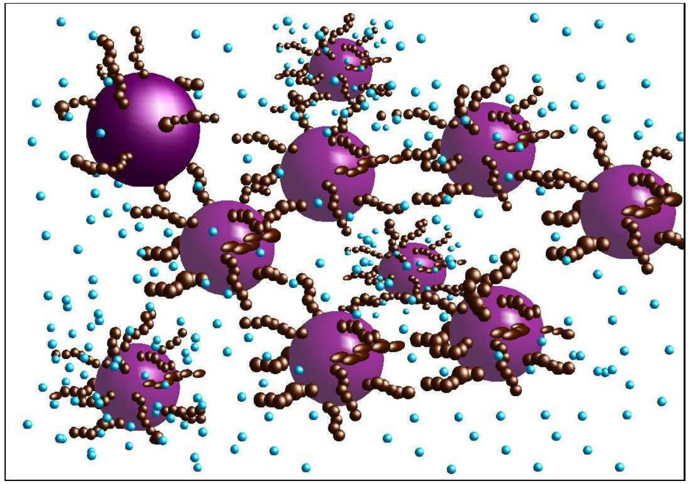
- What is ferrofluid?
- What is magneto-convection?
- Potential applications.
- Problem definition and approach.
- Stability and energy analyses.
- Physical mechanisms discovered.
- Open questions of rheology.

Natural magneto-polarisable fluids:

Fluid	Magnetic susceptibility
Diamagnetic protein solutions (Lysozime)	$\chi \sim -10^{-5}$
Paramagnetic melts or solutions ($MnCl_2$)	$\chi \sim 10^{-4} - 10^{-3}$

Ferro-magnetic fluid (ferro-colloid):

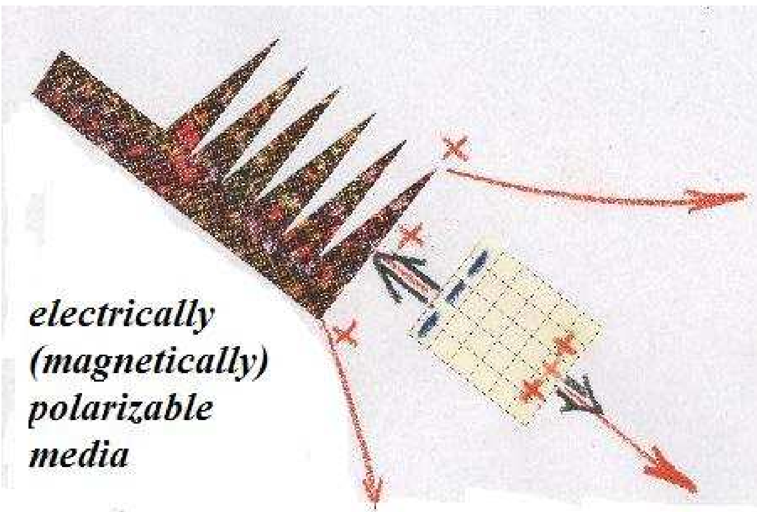
Base: kerosene or mineral oil;
Solid magnetic phase: single domain magnetite, iron or cobalt particles (size $\sim$10 nm, concentration $\sim 10\%$);
Surfactant: mono-layer of oleic acid.



$\chi \sim 5$

Nature of magnetic ponderomotive (Kelvin) force

Electrostatic analogy:



Magnetic force:

$$\vec{F} = M \nabla H \,,$$

M—magnetisation,

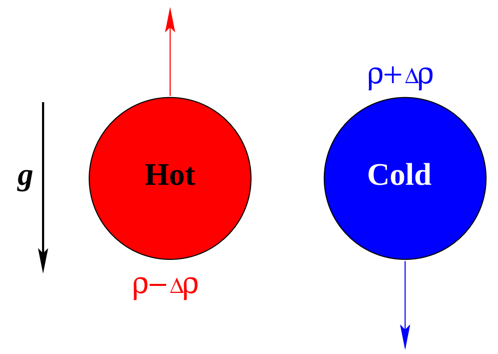
H—magnetic field

Stronger magnetised particles are forced to regions with a stronger magnetic field.

Two types of convection

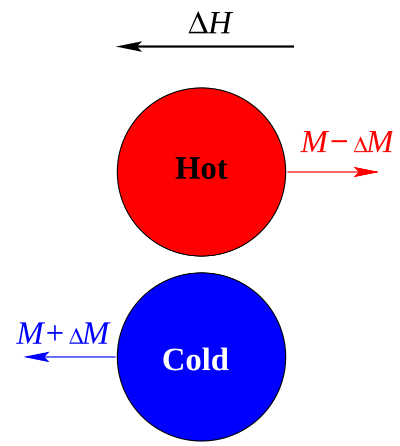
Gravitational convection:

- Local heating leads to fluid expansion $\Delta\rho = -\beta\Delta T$;
- Less dense fluid rises due to buoyancy.



Magnetoconvection:

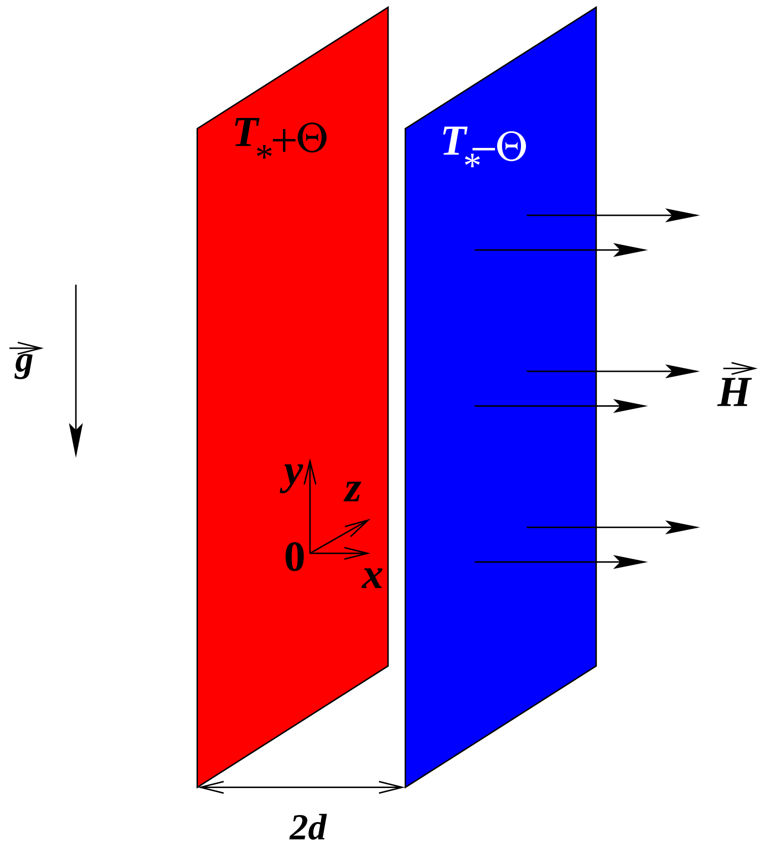
- Local heating leads to partial demagnetisation $\Delta M = -K\Delta T$;
- Stronger magnetised fluid is driven to regions with stronger magnetic field.



Applications:

- Targeted drug delivery in cancer treatment
- Magnetic sealing of joints and gaps
- Cooling of high-power loud-speakers
- Heat exchangers in low gravity conditions (spacecrafts)
- Convection control in protein crystal growth

Geometry sketch:



Non-dimensional governing equations:

$$\frac{\partial \vec{v}}{\partial t} + \vec{v}\cdot\nabla\vec{v} = -\nabla P + \nabla^2\vec{v} - Gr\theta\vec{e}_g - Gr_m\theta\nabla H\,,$$

$$\frac{\partial \theta}{\partial t} + \vec{v}\cdot\nabla\theta = \frac{1}{Pr}\nabla^2\theta\,, \quad \nabla\cdot\vec{v} = 0\,, \quad \nabla\times\vec{H} = \vec{0}\,,$$

$$(1+\chi_*)\nabla\cdot\vec{H} + (\chi-\chi_*)\nabla H\cdot\vec{e}_* - (1+\chi)\nabla\theta\cdot\vec{e}_* = 0\,,$$

$$\vec{M} = \left[(\chi-\chi_*)(H-N) - (1+\chi)\theta\right]\vec{e}_* + \chi_*\vec{H}\,,$$

$$\vec{e}_g = (0,-1,0)\,, \quad \vec{n} = \vec{e}_* = (1,0,0)$$

Boundary conditions:

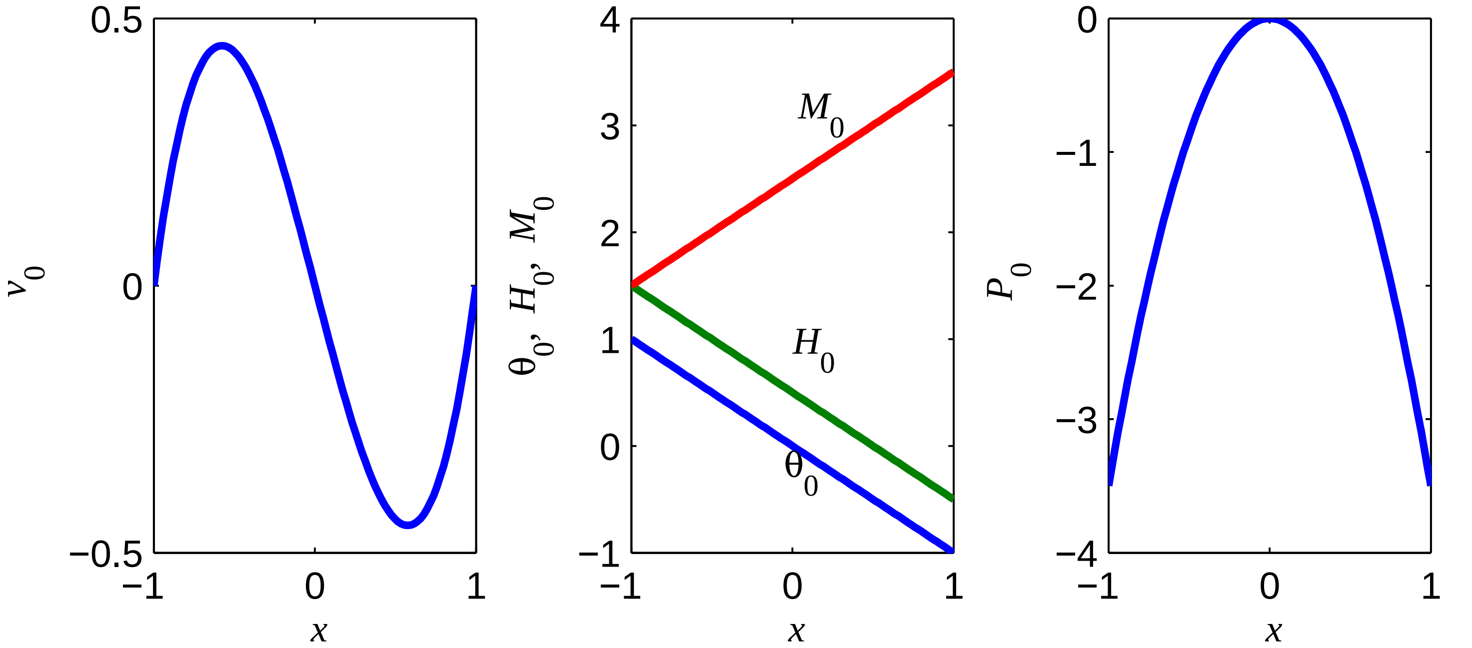
$$\left[\vec{H}^e - \left[(\chi-\chi_*)(H-N) \mp (1+\chi)\right]\vec{e}_* - (1+\chi_*)\vec{H}\right]\cdot\vec{n} = 0\,,$$

$$\vec{v} = \vec{0}\,, \quad \theta = \pm 1\,, \quad \text{at } x = \mp 1$$

Dimensionless parameters:

$$Gr = \frac{\rho_*^2\beta_*\Theta g d^3}{\eta_*^2}\,,\ Gr_m = \frac{\rho_*\mu_0 K^2\Theta^2 d^2}{\eta_*^2(1+\chi)}\,,\ Pr = \frac{\eta_*}{\rho_*\kappa_*}\,,\ N = \frac{H_*(1+\chi)}{K\Theta}$$

Basic flow solutions:

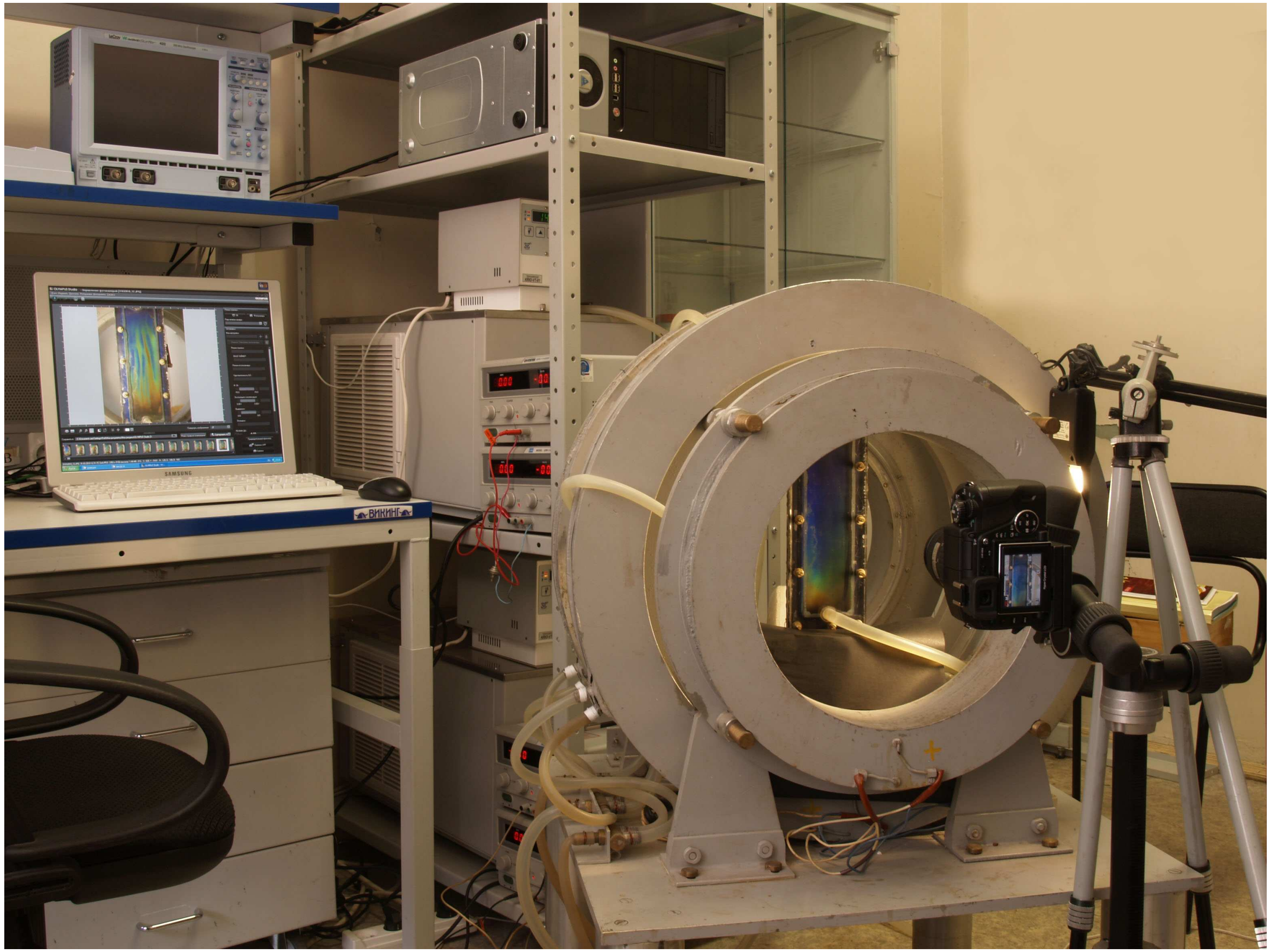


Typical parameter ranges:

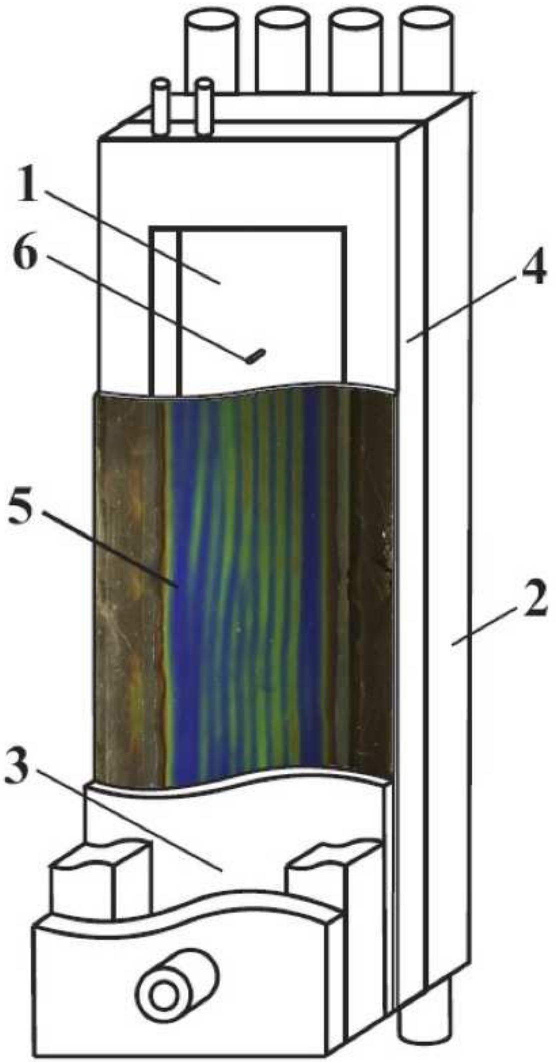
$$Gr \sim 0 - 8\,, \quad Gr_m \sim 0 - 15\,, \quad Pr \sim 130\,, \quad \chi \sim 5\,, \quad \chi_* \sim 5$$

- Uniform external magnetic field results in a nonuniform field inside the layer;
- Stronger magnetised fluid is in a weak magnetic field region.

Experimental equipment:

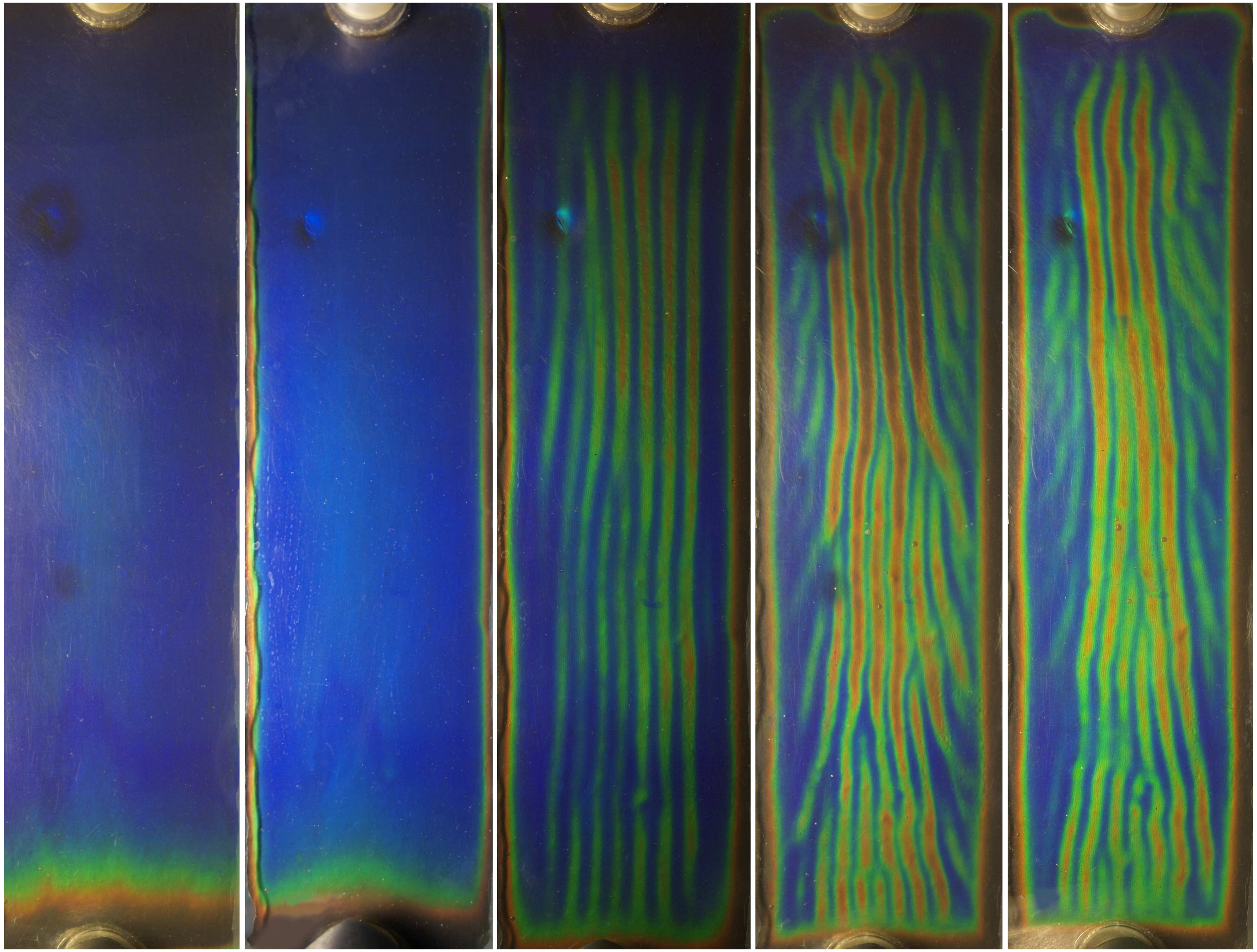


Experimental convection chamber:



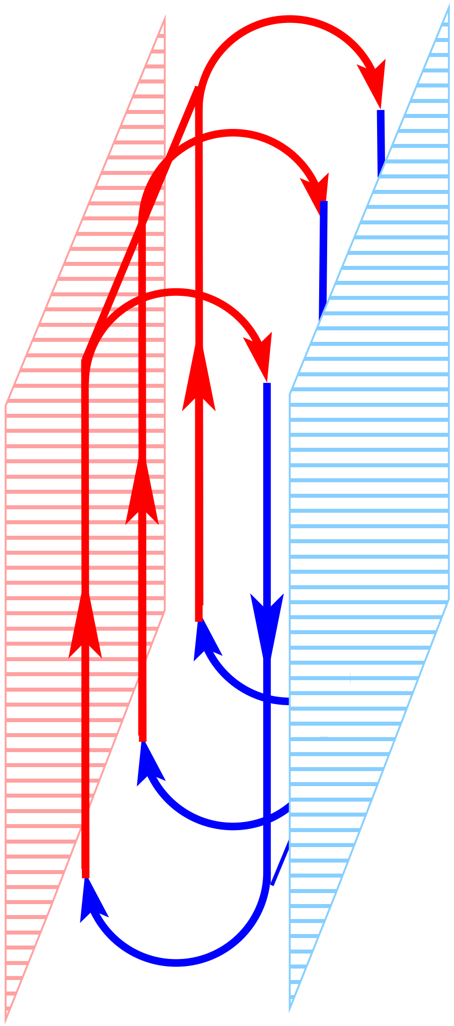
1. Ferrofluid layer
2. Copper heat exchanger
3. Plexiglas heat exchanger
4. Plexiglas frame
5. Thermo-sensitive liquid crystal sheet
6. Thermocouples

Experimentally observed instability in uniform transverse magnetic field:

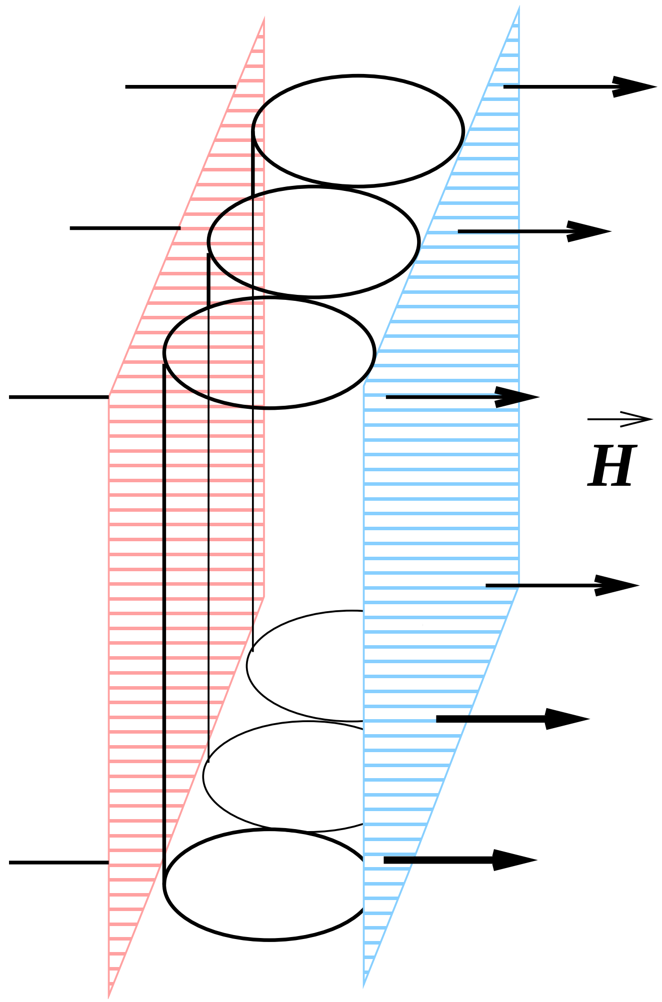


Schematic fluid motion:

Basic buoyancy-induced flow:



Flow in magnetic field:



Linearised perturbation equations (normal mode form):

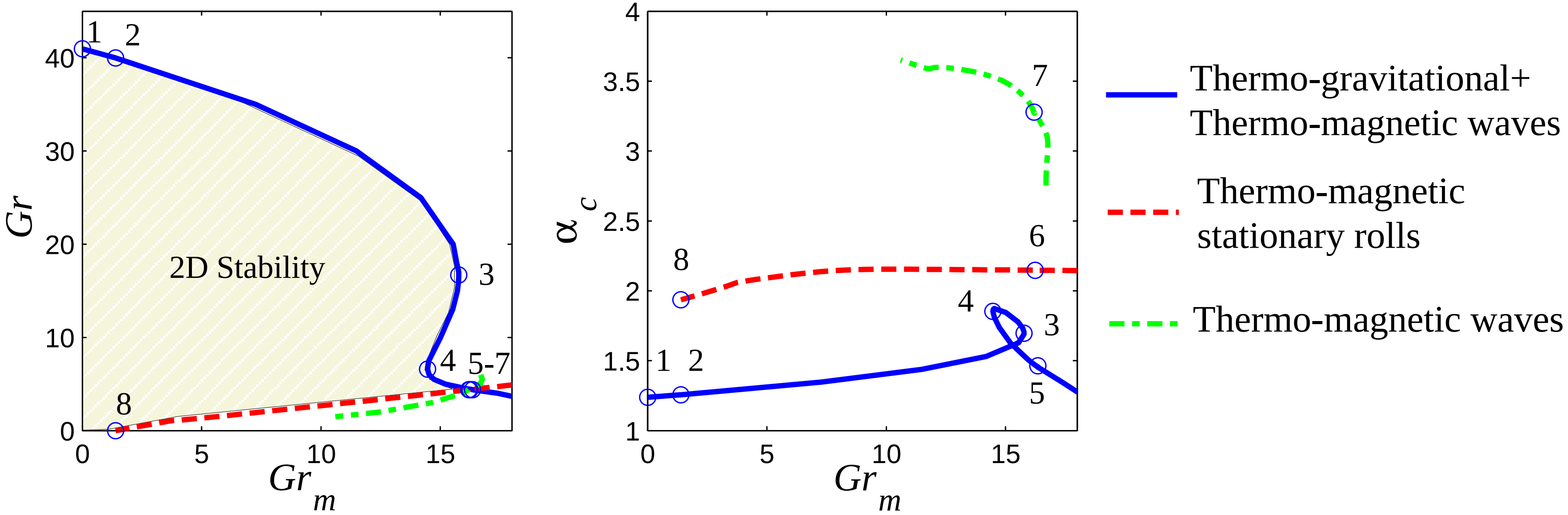
$$
\begin{aligned}
\sigma u &+ \left(\alpha^2 + i\alpha v_0 - D^2\right) u + DP + Gr_m DH_0\,\theta + Gr_m \Theta_0 D^2\phi = 0\,,\\
\sigma v &+ Dv_0\, u + \left(\alpha^2 + i\alpha v_0 - D^2\right) v + i\alpha P - Gr\theta + i\alpha Gr_m \Theta_0 D\phi = 0\,,\\
\sigma\theta &+ D\Theta_0 u + \left(\frac{\alpha^2 - D^2}{Pr} + i\alpha v_0\right)\theta = 0\,,\\
& Du + i\alpha v = 0\,, \quad \left(D^2 - \frac{1+\chi_*}{1+\chi}\alpha^2\right)\phi - D\theta = 0\,,\\
\sigma &= \sigma^R + i\sigma^I \text{— complex amplification rate}\,,\\
\alpha &\text{ — disturbance wavenumber}
\end{aligned}
$$

Boundary conditions:

$$
u_1 = v_1 = w_1 = \theta_1 = 0\,, \quad (1+\chi)D\phi_1 \pm \sqrt{\alpha^2 + \beta^2}\phi_1 = 0 \quad \text{at } x = \pm 1\,,
$$

where $D \equiv d/dx$ and $\vec{H} = (D\phi, i\alpha\phi)$.

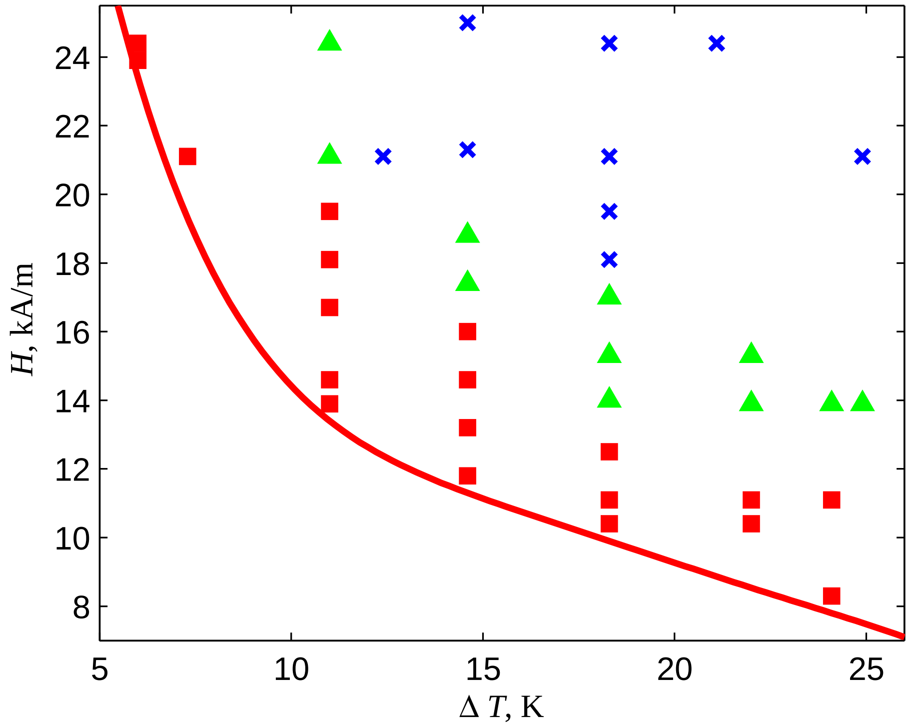
Computational stability results:



- Full 3D map is obtained using inverse Squire's transformation (Suslov, *Phys. Fluids*, 2008).

How does this compare with experiment?

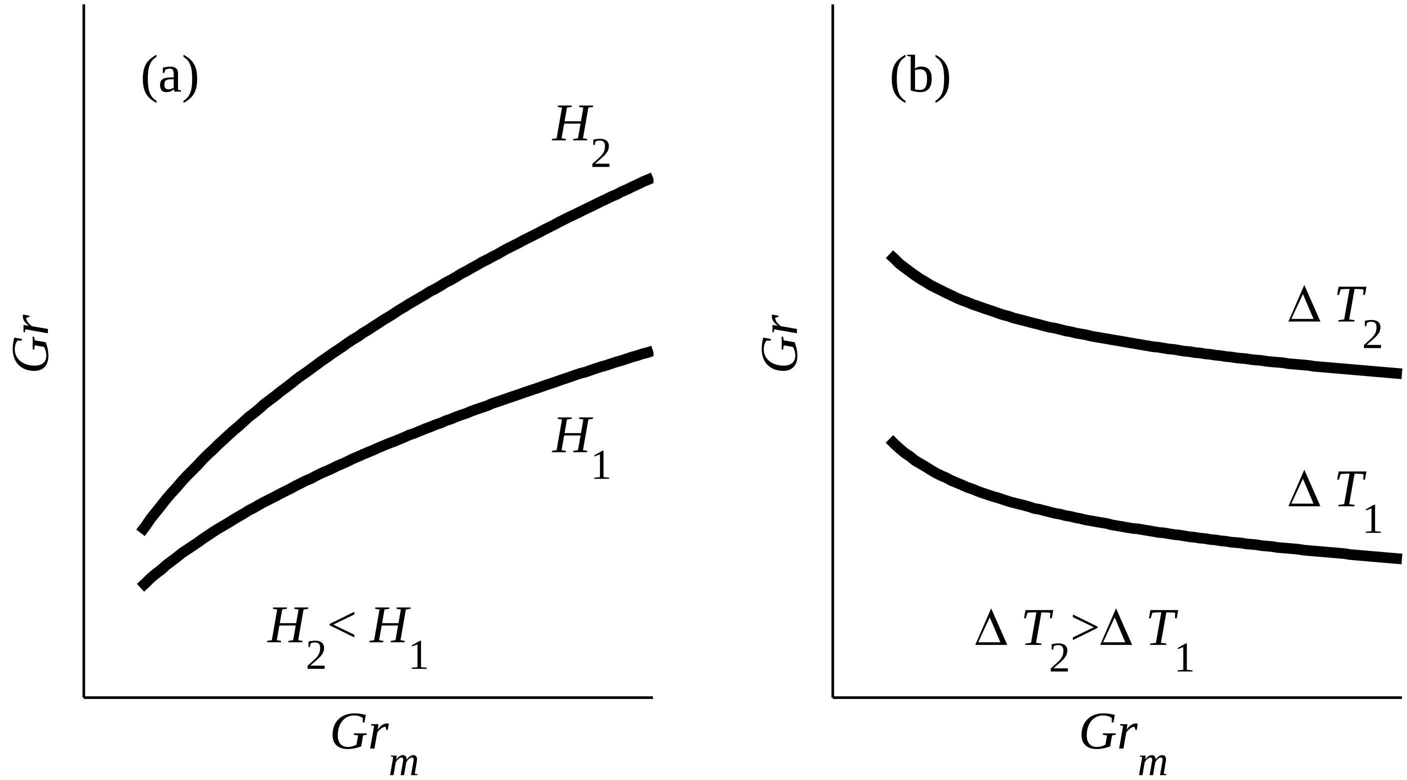
Experimental stability map:



- Suslov et al, *Phys. Rev. E*, 2012.

Need to convert H and ΔT to $Gr = \frac{\rho_*^2 \beta_* \Theta g d^3}{\eta_*^2}$ and $Gr_m = \frac{\rho_* \mu_0 K^2 \Theta^2 d^2}{\eta_*^2 (1+\chi)}$.

Two experimental scenarios



- Viscosity is not constant and is not known.
- Accurate determination of Gr and Gr_m is currently impossible.

Practical design problems!

Disturbance energy balance equation:

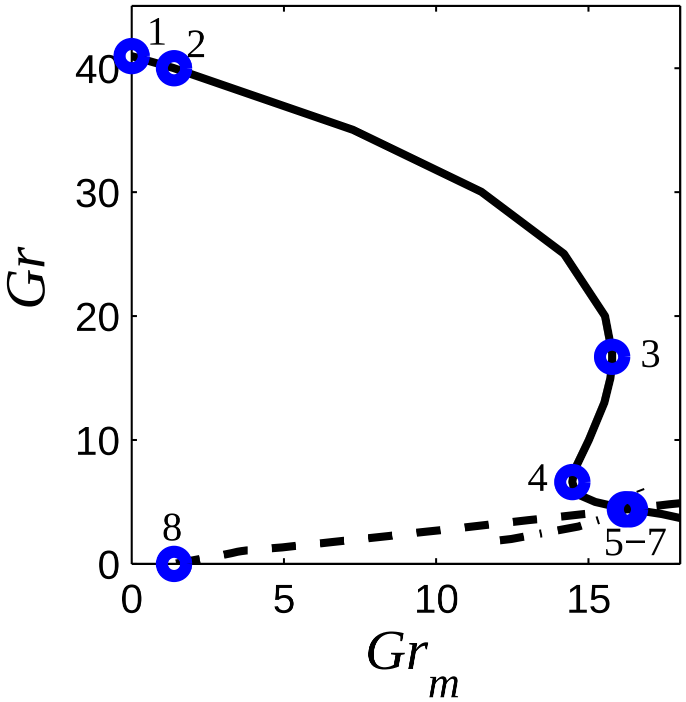
$$\sigma^R \Sigma_k = \Sigma_{uv} + \Sigma_{m1} + \Sigma_{m2} + \Sigma_{Gr} + \Sigma_{vis} \, ,$$

$$\Sigma_k = \int_{-1}^{1} \left(|u|^2 + |v|^2 \right) \, dx > 0 \, , \quad \Sigma_{uv} = -\int_{-1}^{1} Dv_0 \Re(u\bar{v}) \, dx \, ,$$

$$\Sigma_{m1} = \int_{-1}^{1} \underbrace{-Gr_m DH_0 \, \Re(\theta \bar{u})}_{E_{m1}} \, dx \, , \quad \Sigma_{m2} = \int_{-1}^{1} \underbrace{Gr_m D\theta_0 \, \Re(D\phi \, \bar{u})}_{E_{m2}} \, dx \, ,$$

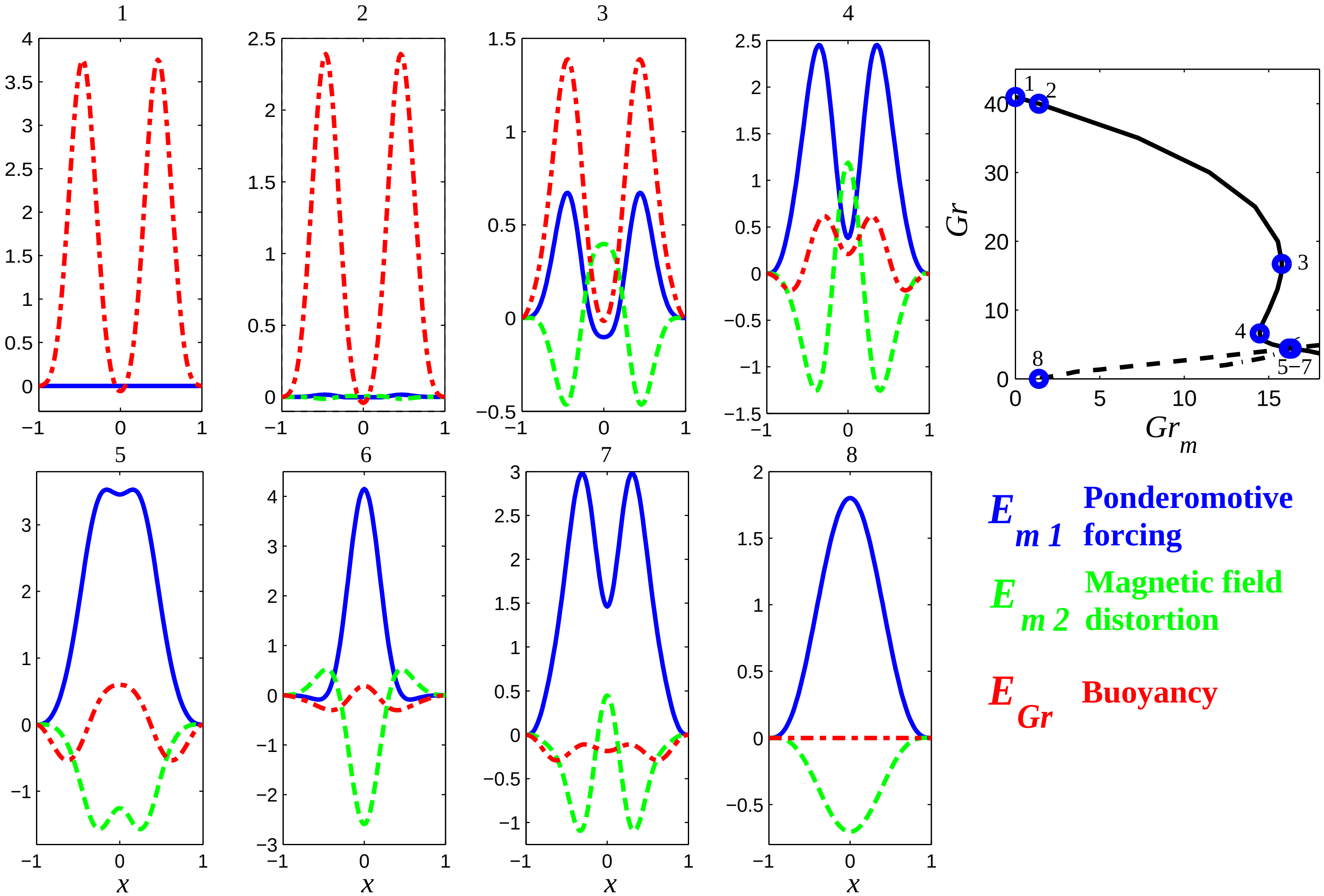
$$\Sigma_{Gr} = \int_{-1}^{1} \underbrace{Gr \Re(\theta \bar{v})}_{E_{Gr}} \, dx \, , \quad \Sigma_{vis} = -\alpha^2 E_k - \int_{-1}^{1} \left(|Du|^2 + |Dv|^2 \right) \, dx = -1 \, .$$

Disturbance energy integrals for selected marginal stability points:



	Σ_{uv}	Σ_{m1}	Σ_{m2}	Σ_{Gr}
1	-0.006	0	0	1.006
2	-0.007	0.004	-0.002	1.005
3	-0.011	0.302	-0.086	0.795
4	-0.003	1.141	-0.318	0.180
5	0.001	1.795	-0.759	-0.036
6	0.002	1.812	-0.595	-0.219
7	-0.003	1.516	-0.350	-0.163
8	0	1.584	-0.584	0

Disturbance energy integrands:

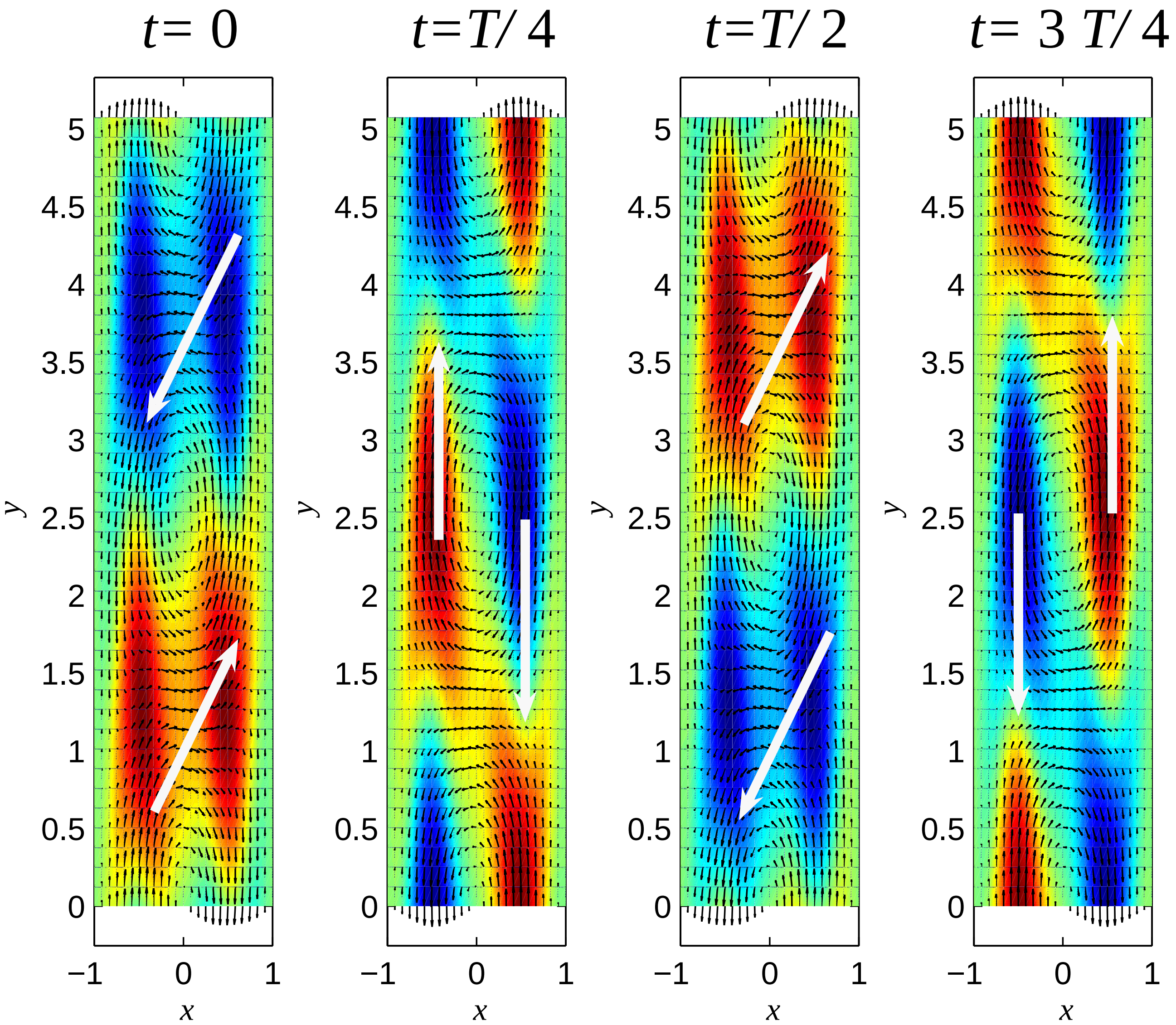


$E_{m\,1}$ Ponderomotive forcing

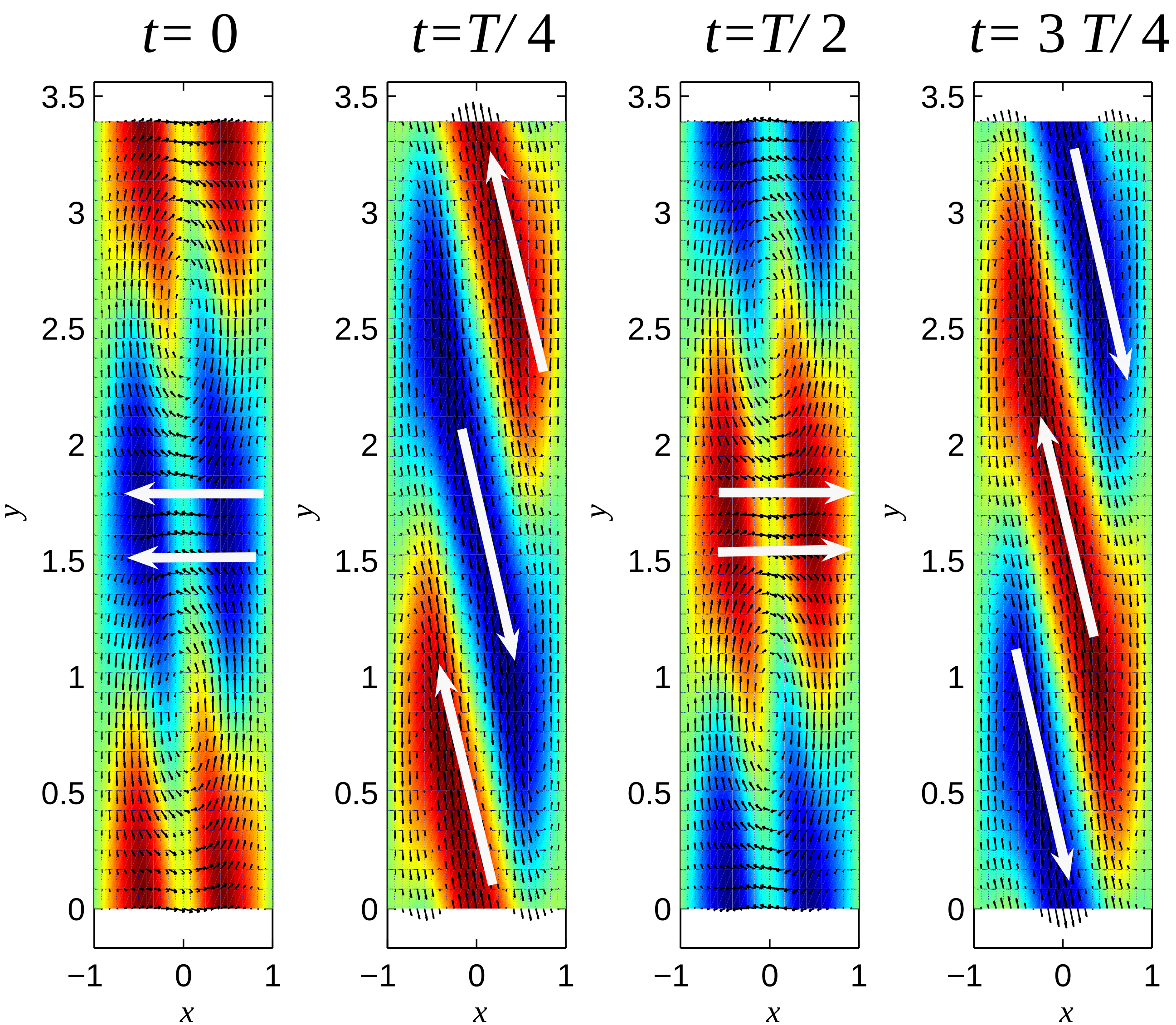
$E_{m\,2}$ Magnetic field distortion

E_{Gr} Buoyancy

Thermo-gravitational waves (point 1):



Thermo-magnetic waves (point 4):



Summary:

- The instability in a vertical ferro-fluid layer is caused by two physical mechanisms: thermo-gravitational (buoyancy) and thermo-magnetic.
- Three instability patterns are found: counter-propagating thermal waves (large Gr, small Gr_m), (new) counter-propagating thermo-magnetic waves (large Gr_m, intermediate Gr) and stationary magneto-convection rolls (intermediate to large Gr_m, small Gr).
- The propagating thermal or thermo-magnetic instability waves form horizontal or inclined convection rolls; stationary magneto-convection rolls remain vertical.
- Knowledge of rheological properties of ferrofluids is crucial for practical design of applications.

Surprise, surprise: