The Schlichting boundary layer under SAW

Pattern formation and contact line displacement

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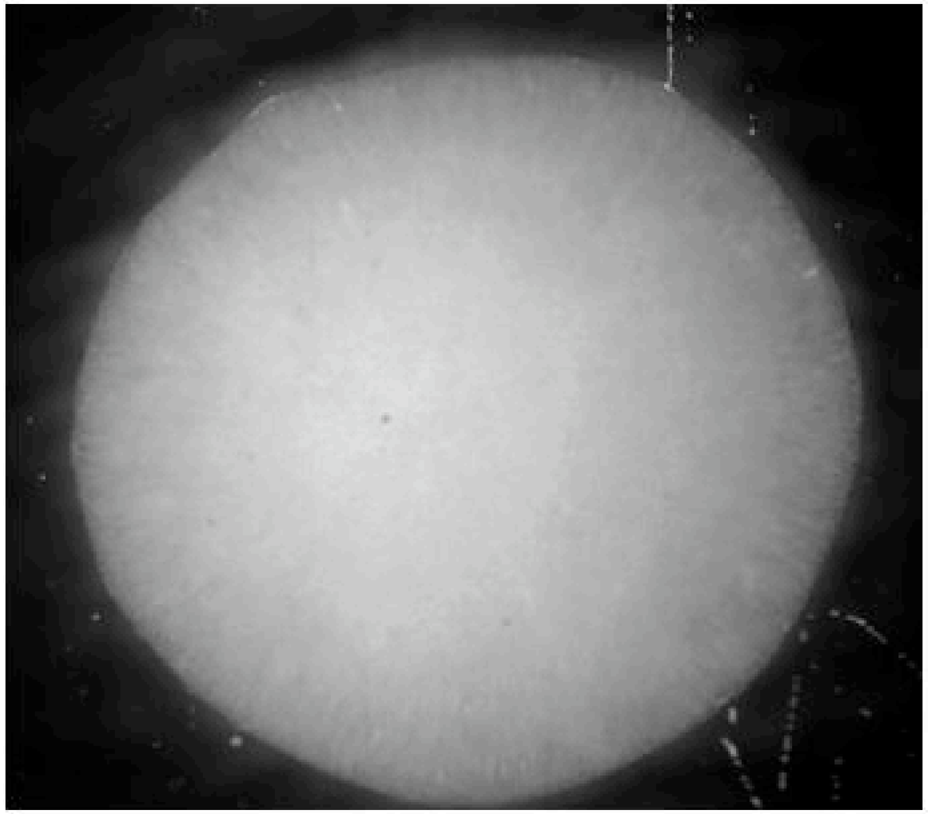
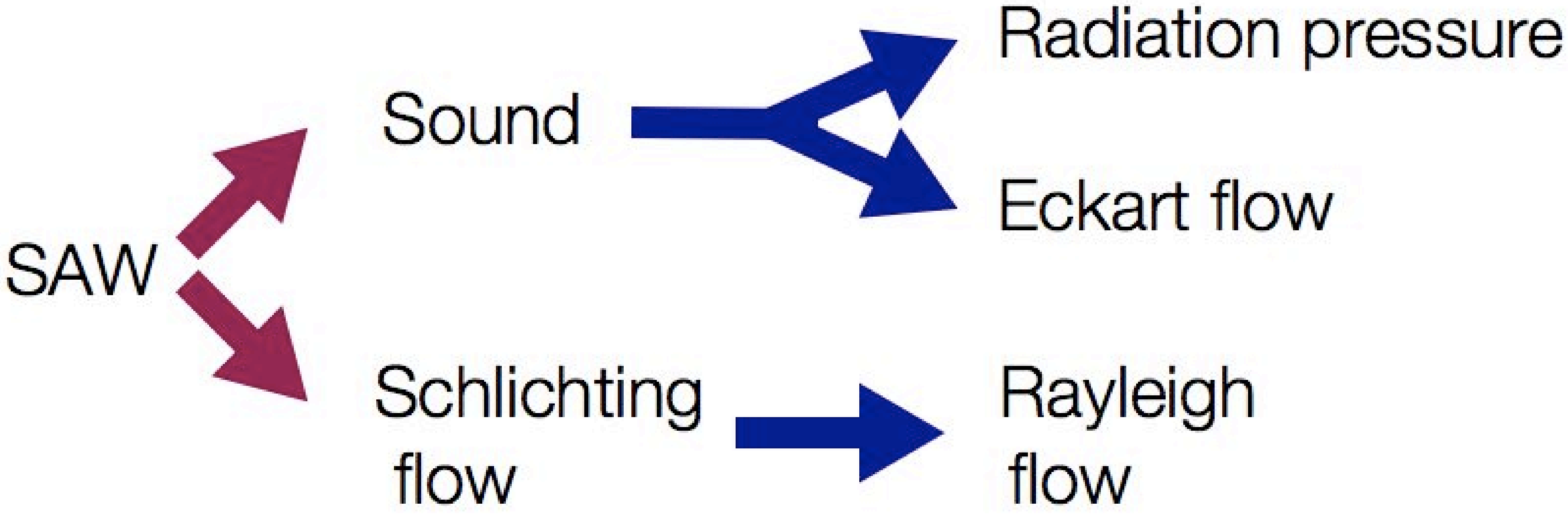


MONASH University
Engineering


SAW

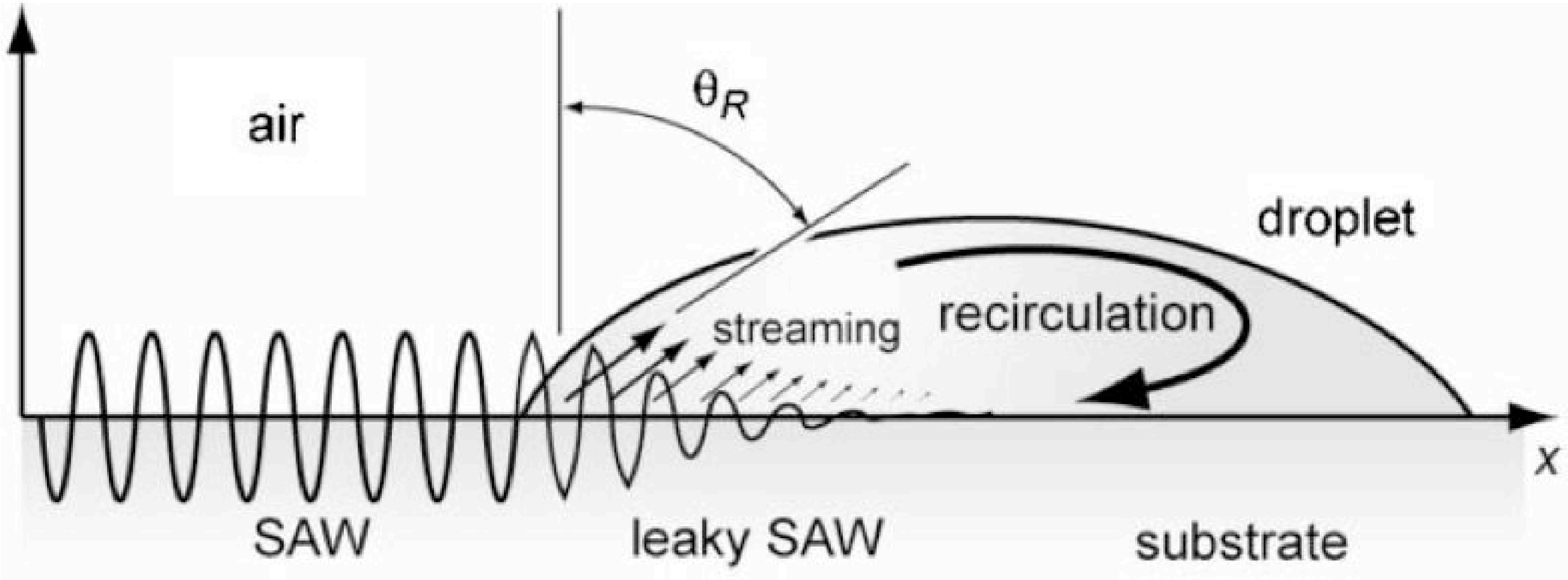
Sound

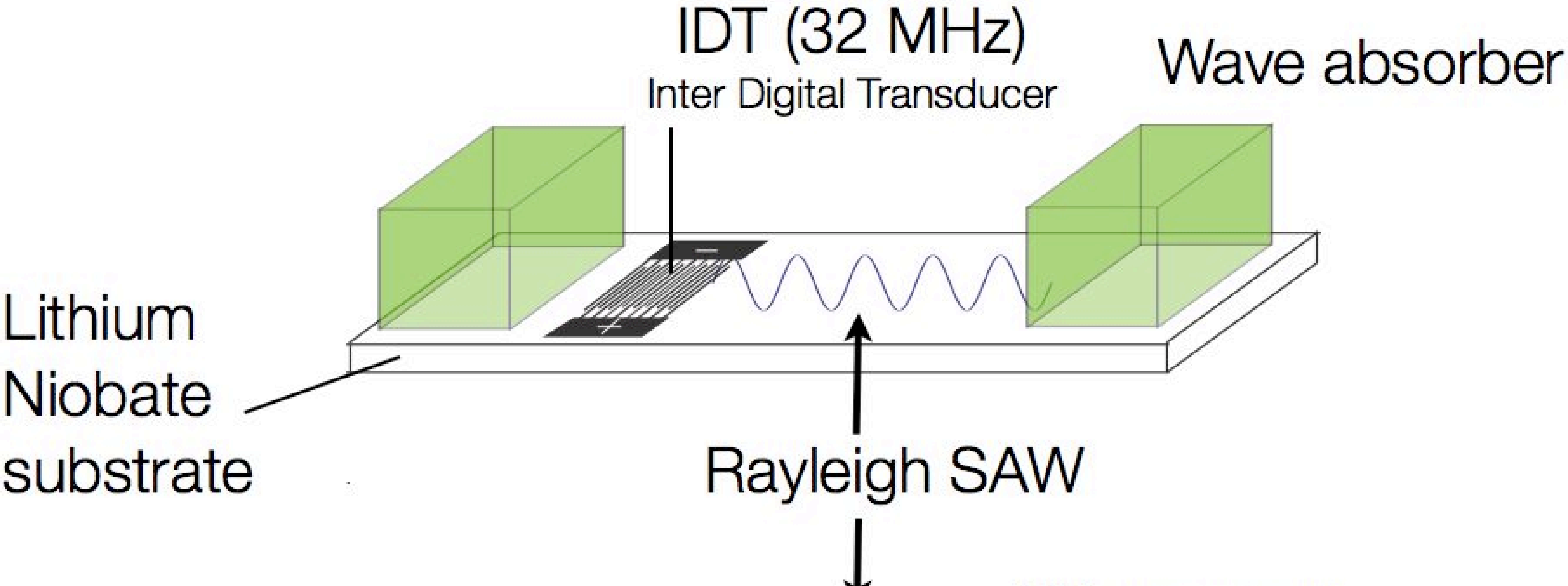
Radiation pressure

Eckart flow

Schlichting flow

Rayleigh flow





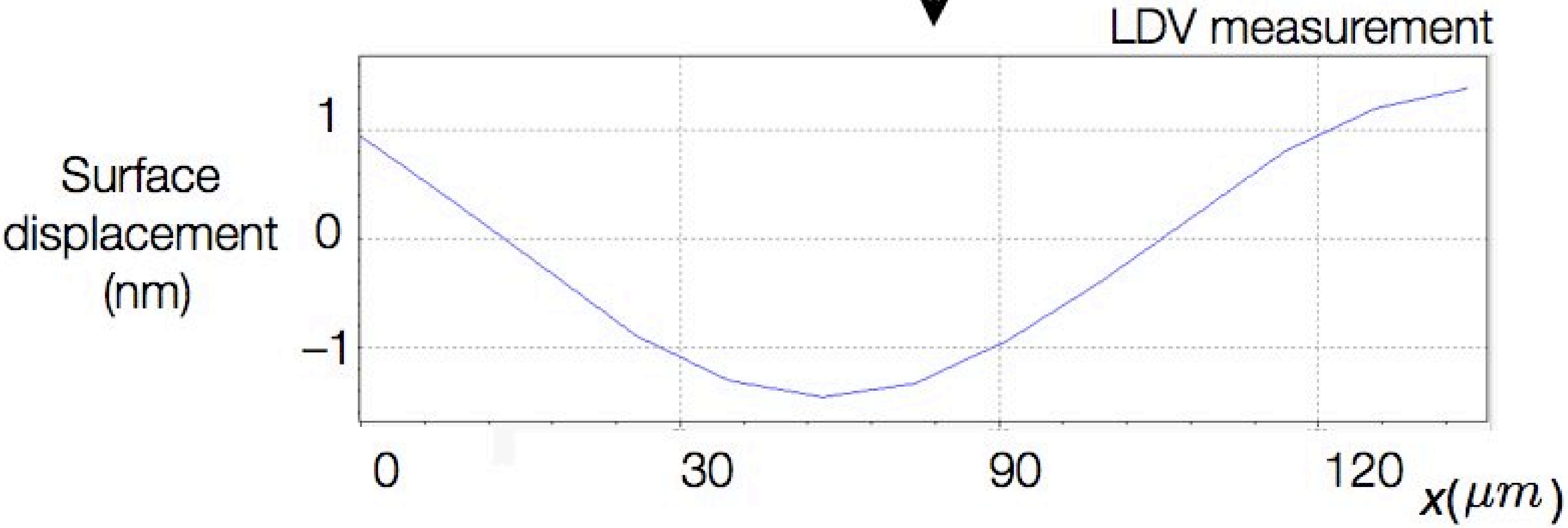
IDT (32 MHz)

Inter Digital Transducer

Wave absorber

Lithium Niobate substrate

Rayleigh SAW



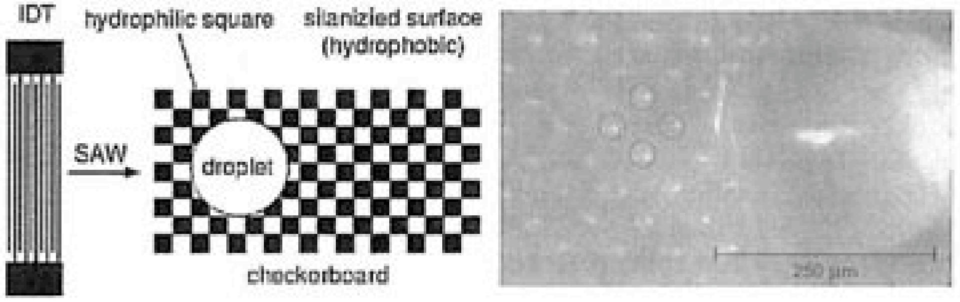
SAW Device

Propagating SAW

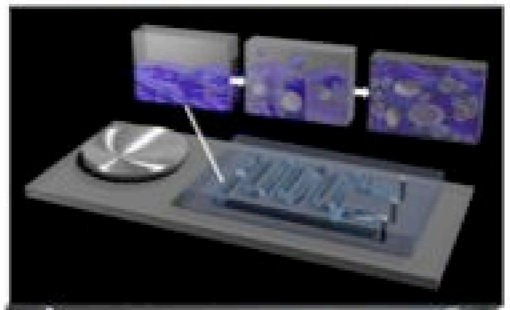
Background

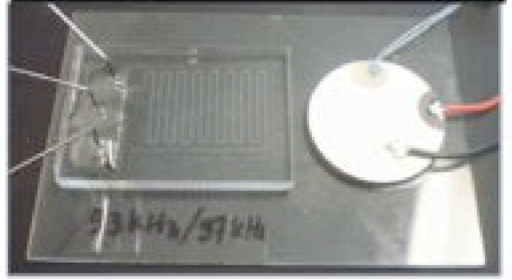
SAW induced microfluidics

Wixforth et al, 2004
Scortesse et al, 2002
Liquid Patterning

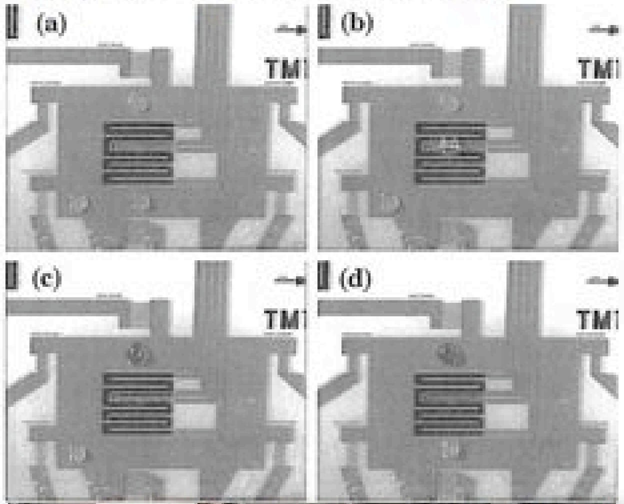


Tandiono et al, 2010
(microfluidic cavitation)

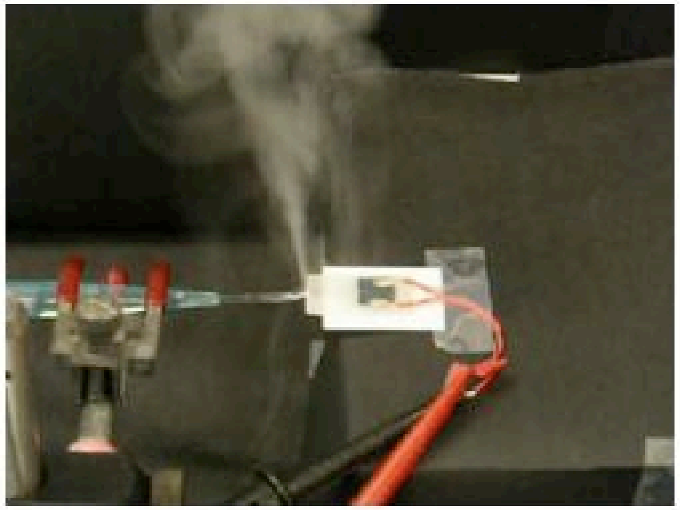




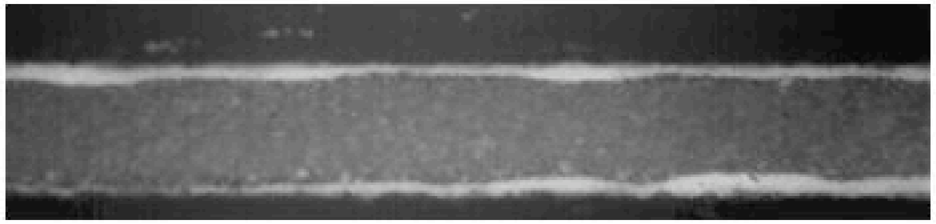
Alzuaga et al, 2005
Daniel et al, 2005
Wixforth et al, 2004
Renaudin et al, 2005
(drop translation)



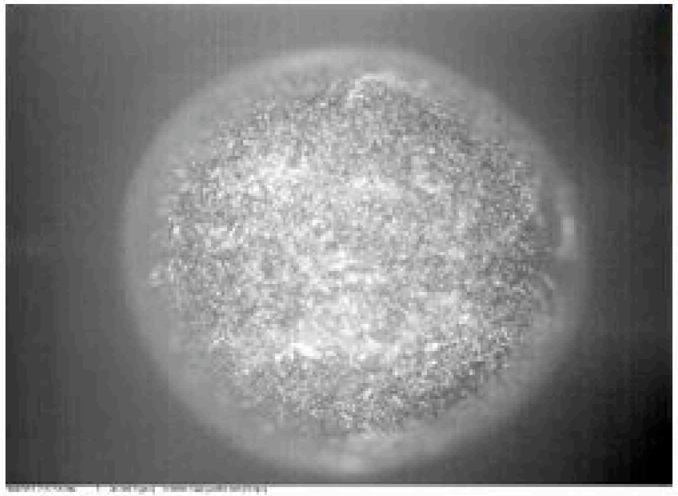
Qi et al, 2008;2010
{D. Collins (MNRL)}
(Atomization)



Tan et al, 2007
Girardo et al, 2008
(pumping fluid in
micro channels)

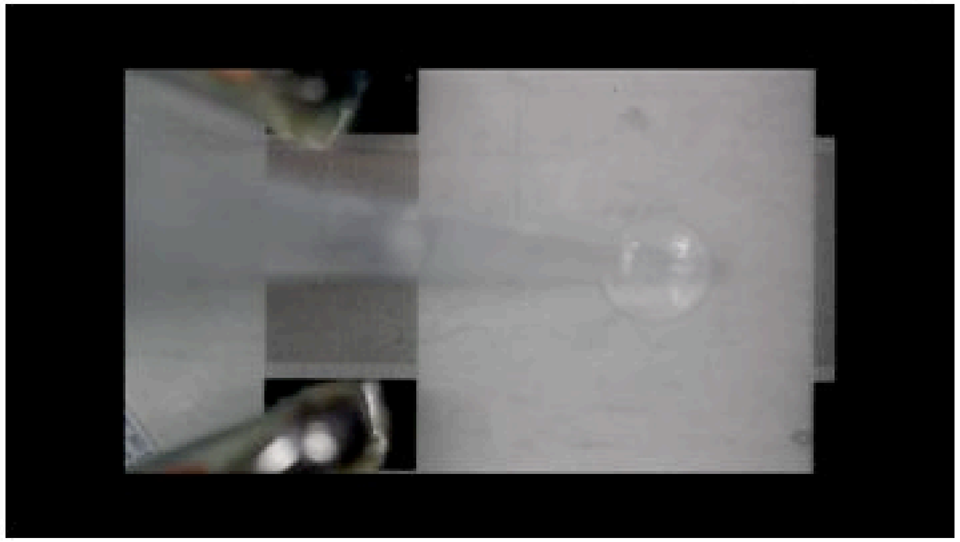


Tan et al, 2007
Dorrestijn et al, 2007
{M. Dentry (MNRL)}
(Particle separation)

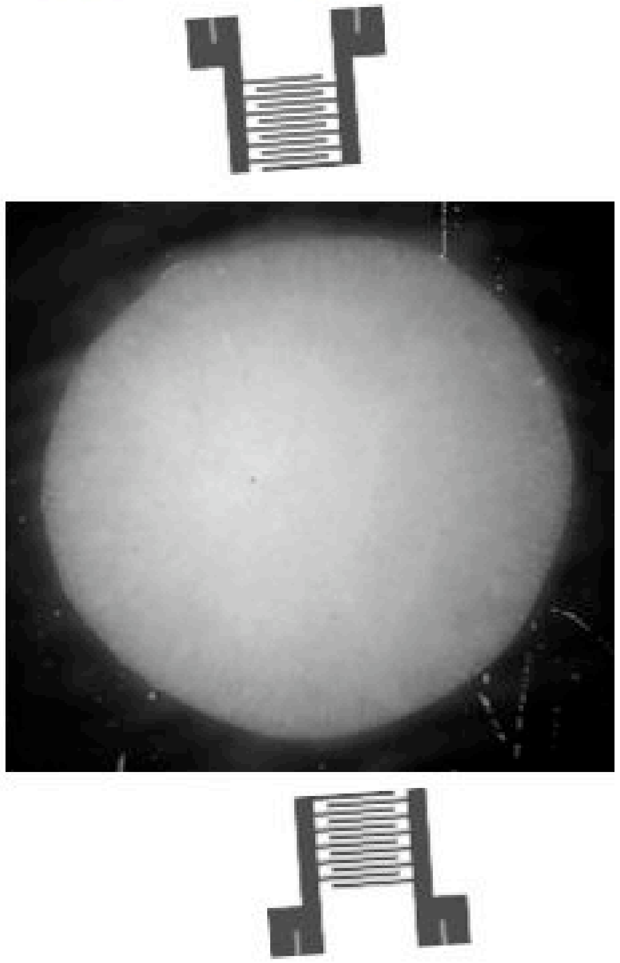


Spreading films

A. Rezk (MNRL)

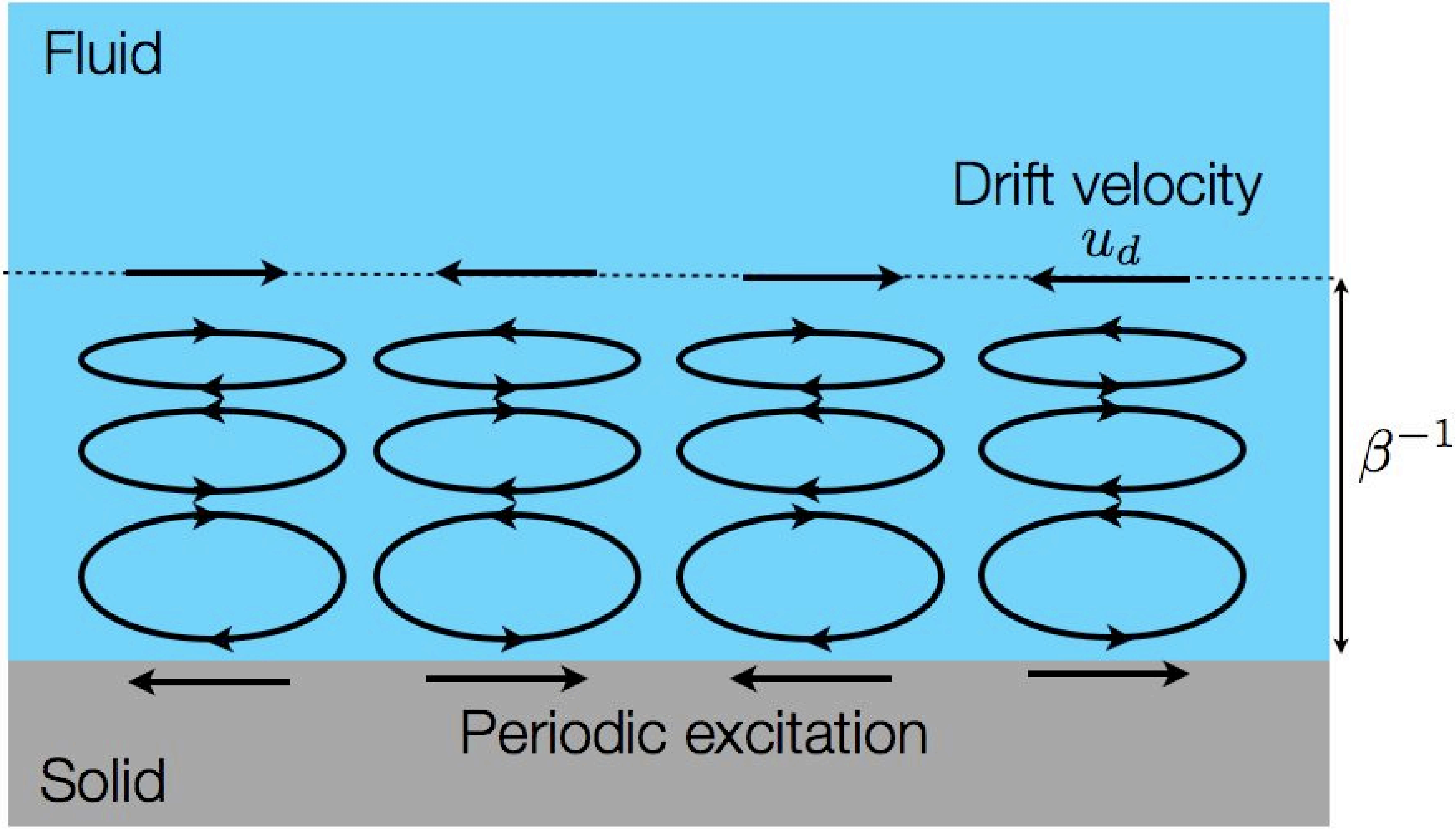


Aggregate patterning



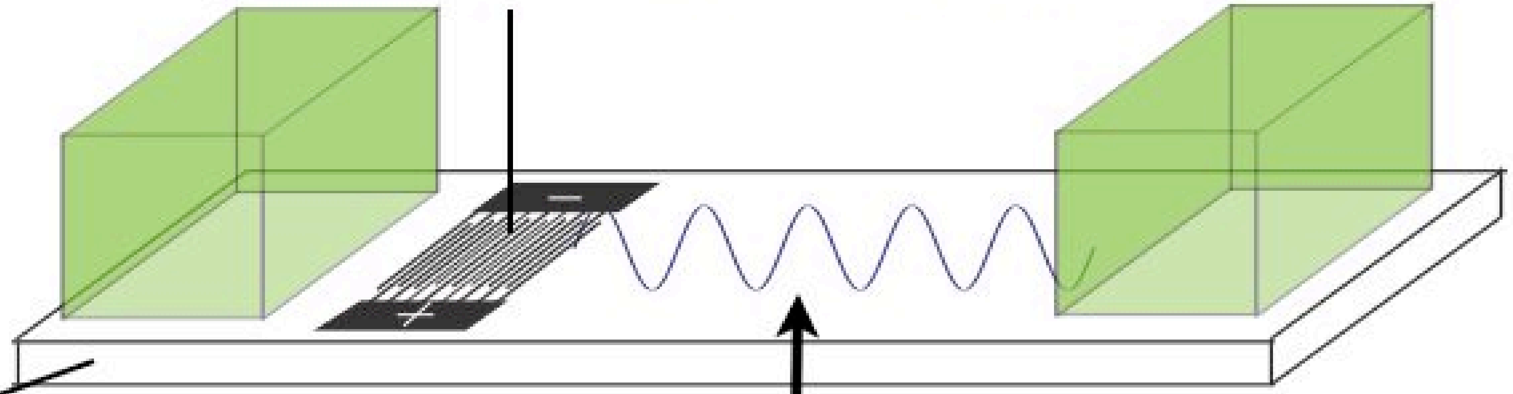
Applications:

- Drop microfluidics
- Micro electronics
- Particle separation
- Particle concentration



Sketch

Schlichting boundary layer



IDT (32 MHz)

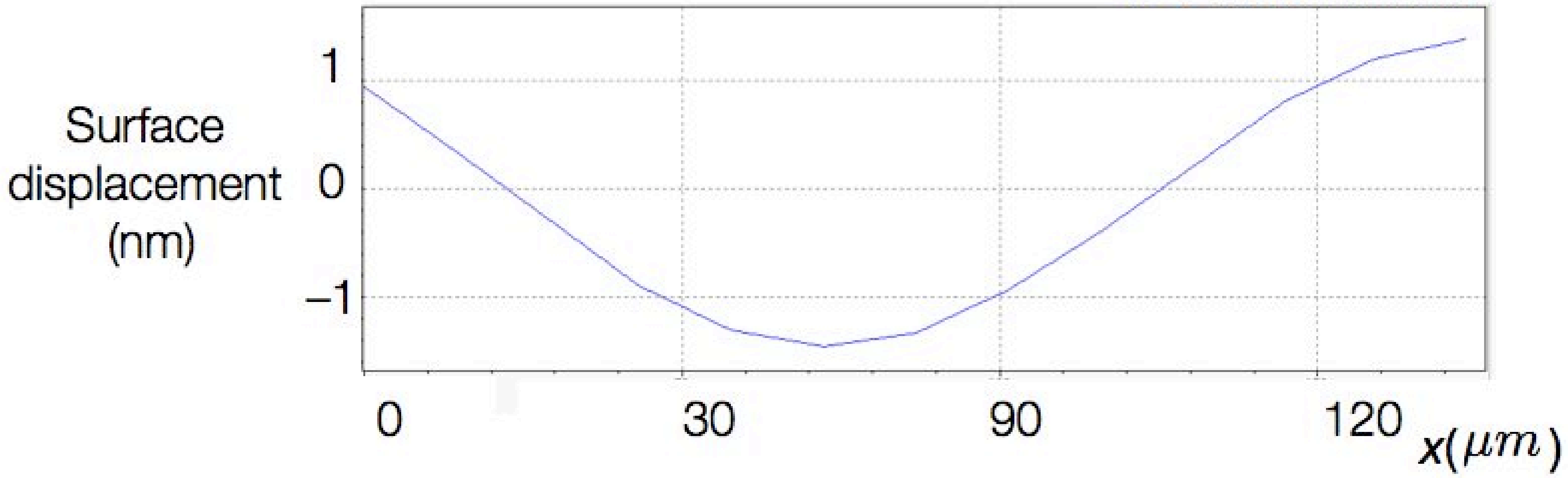
Inter Digital Transducer

Wave absorber

Lithium Niobate substrate

Rayleigh SAW

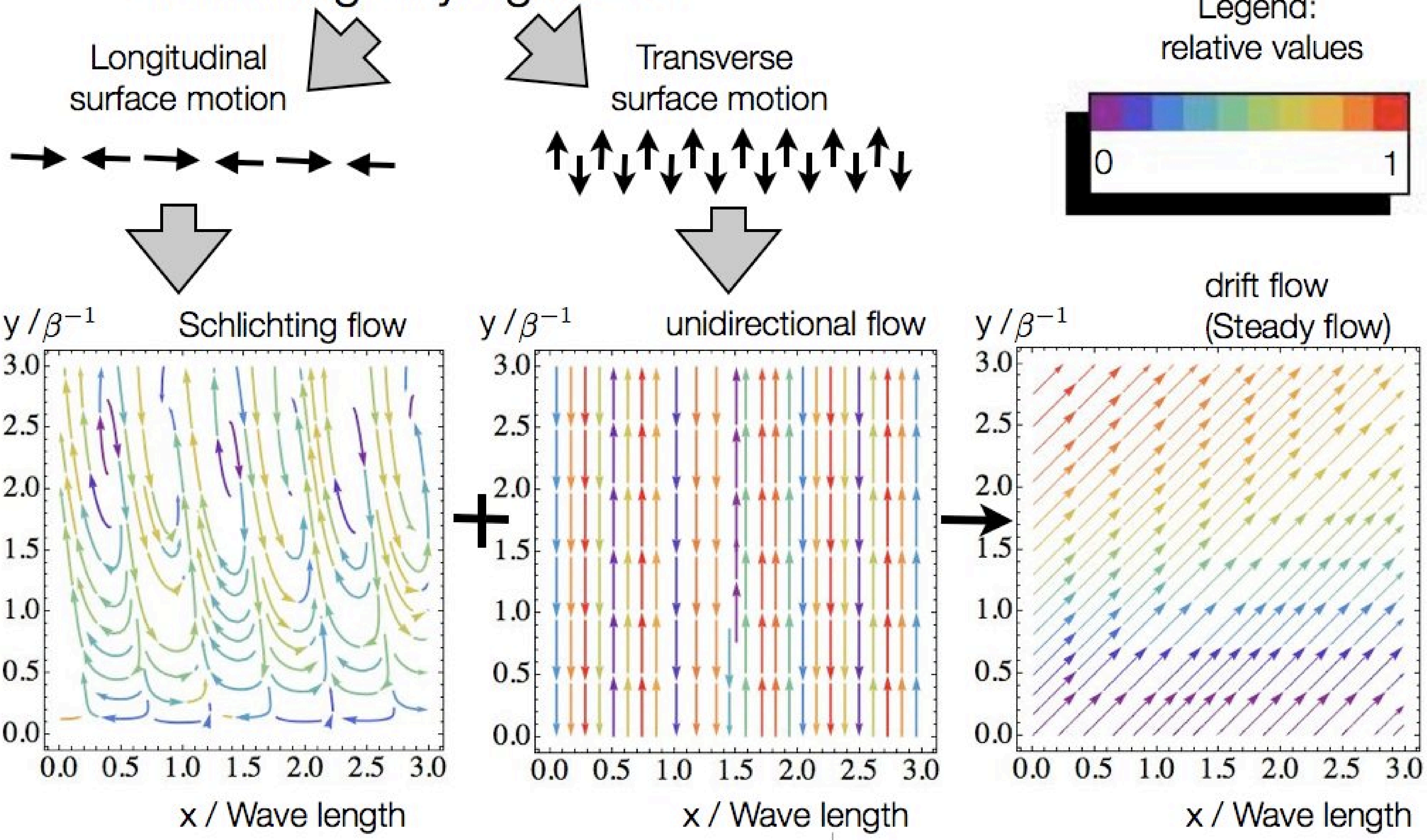
LDV measurement



SAW Device

Propagating SAW

Advancing Rayleigh SAW



Longitudinal surface motion

Transverse surface motion

Legend: relative values

Schlichting flow

unidirectional flow

drift flow (Steady flow)

Stream lines

Schlichting boundary layer

β^{-1}: Characteristic thickness

Dimensionless quantities

$\eta \equiv \beta^{-1}/k_{SAW}^{-1} \ll 1$ (boundary layer thickness) / (SAW wave length)

$\epsilon \equiv U/C_{SAW} \ll 1$ Mach number: (Flow velocity) / (Sound speed)

Scaling

$$t \to t/\omega,\ x \to k^{-1}x,\ y \to \beta^{-1}y,\ u_x \to U u_x,$$
$$u_y \to \eta U u_y,\ \alpha \to k\alpha,\ \psi \to \beta^{-1} U \psi$$

$$\beta^{-1} \equiv \sqrt{2\mu/\rho\omega}$$

Stream function equation

$$\frac{1}{2}\partial_y^4\psi = \partial_t\left(\partial_y^2\psi\right) + \epsilon\left[\partial_y\psi\,\partial_x\left(\partial_y^2\psi\right) - \partial_x\psi\,\partial_y\left(\partial_y^2\psi\right)\right] + \mathcal{O}(\eta^2)$$

B.C.s: Solid/liquid: **SAW** or **No SAW**

$$\begin{pmatrix} u_x \\ u_y \end{pmatrix}_{y=0} = \begin{pmatrix} \chi e^{i(t-x)-\alpha x} \\ \eta^{-1} e^{i(t-x+\varphi)-\alpha x} \end{pmatrix} \qquad \begin{pmatrix} u_x \\ u_y \end{pmatrix}_{y=0} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

Far from the solid: **Standing bulk wave** or **bounded velocity field**

$$u_x|_{\beta y \to \infty} = 2\chi U e^{i\omega t} \cos\left(\delta k x\right) \qquad u_x|_{\beta y \to \infty} < \infty$$

Formalism

Characteristics and Boundary Conditions

Expansion

$$u_x = \sum_n f_n u_{x,n}, \qquad u_y = \sum_n f_n u_{y,n}, \qquad \psi = \sum_n f_n \psi_n,$$

$$\epsilon \ll \epsilon/\eta \ll 1 \ll 1/\eta$$

For water/oil at *O*(1-100MHz) SAW

Stream function components

$O(1/\eta)$: $\frac{1}{2}\partial_y^4 \psi_{-1} = \partial_t \left(\partial_y^2 \psi_{-1}\right)$ .... **Unidirectional flow**

$O(1)$: $\frac{1}{2}\partial_y^4 \psi_0 = \partial_t \left(\partial_y^2 \psi_0\right)$ .... **Classic Schlichting leading order flow**

$O(\epsilon/\eta)$: $\frac{1}{2}\partial_y^4 \left\langle \psi_1 \right\rangle = -\left\langle \partial_x \overline{\psi}_{-1} \partial_y^3 \overline{\psi}_0 \right\rangle$ .... **drift flow**

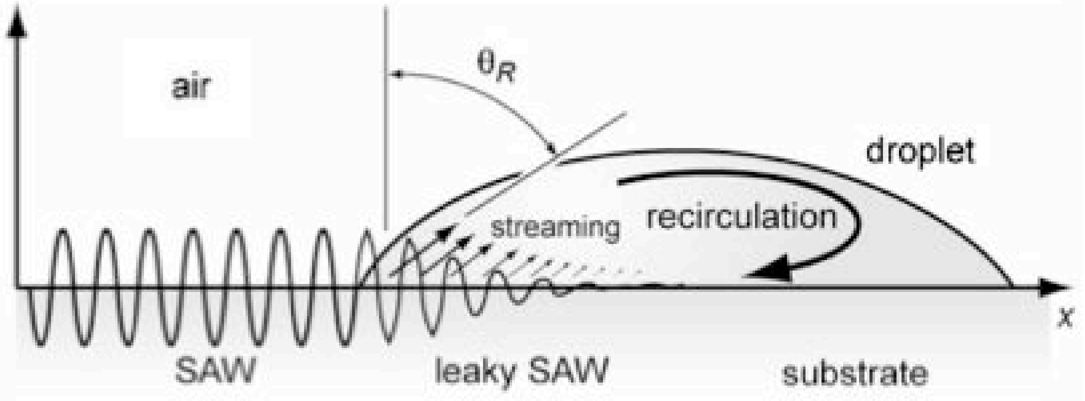
$O(\epsilon)$ .... **Classic Schlichting drift flow**

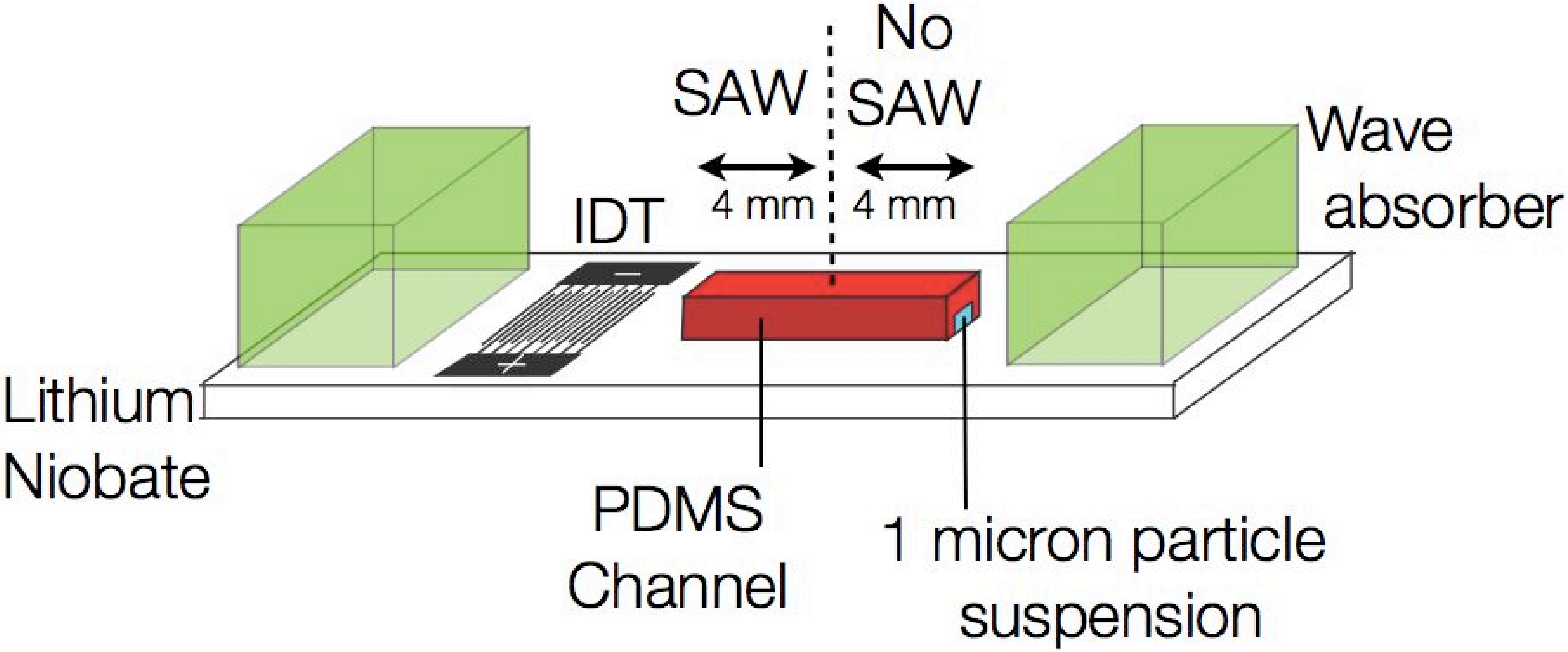
$\langle\rangle$ Averaging over time

$\overline{\psi} \equiv Real(\psi)$

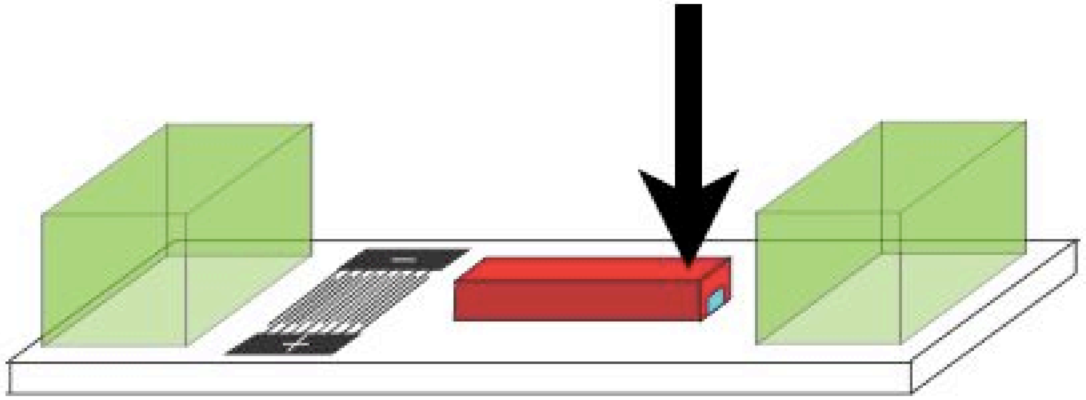
Formalism

Stream function and asymptotic analysis





Sketch of a SAW device



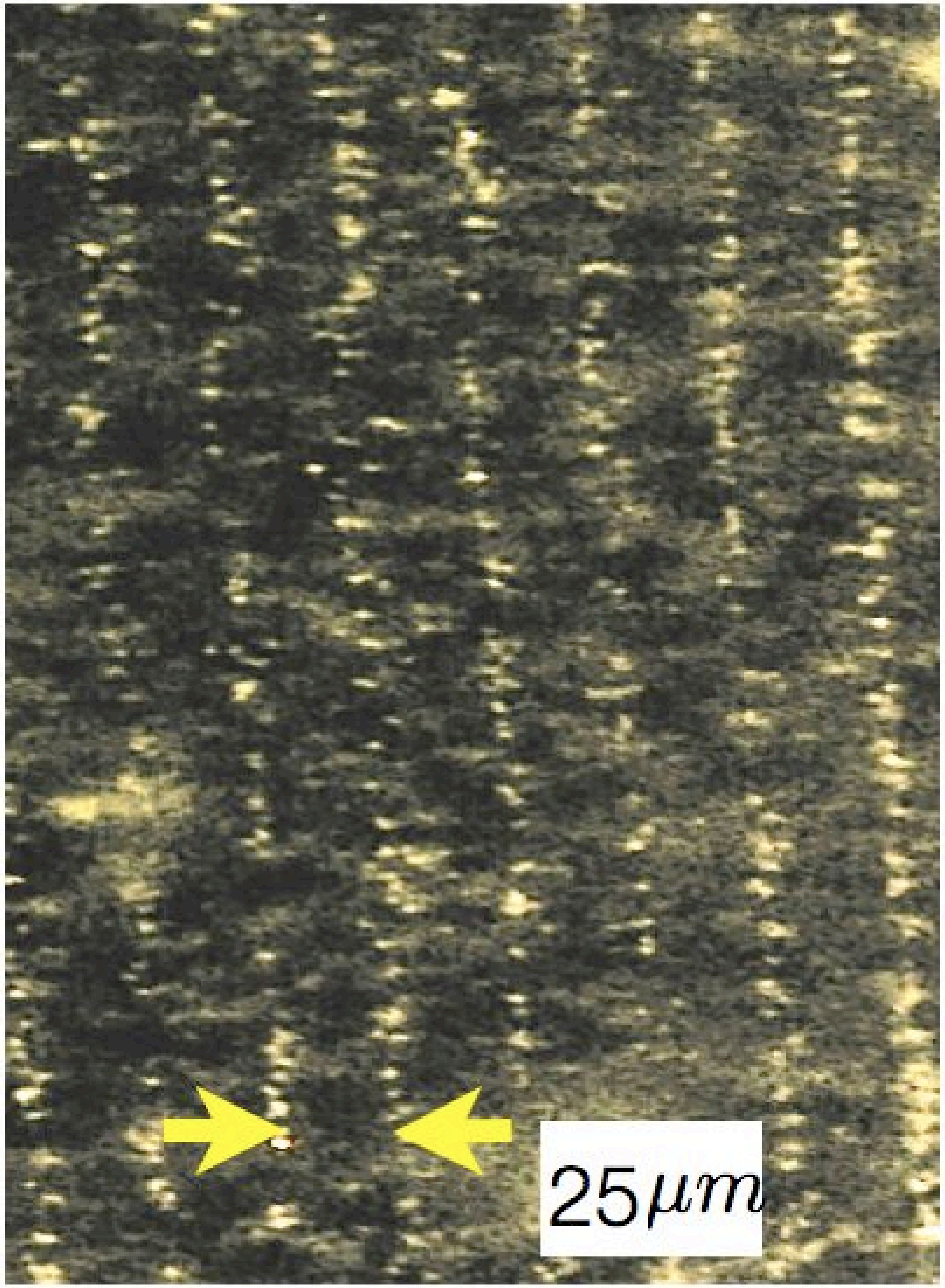
SAW induced Stationary bulk wave in the liquid

+

1 micron particles

↓

Thin aggregate pattern

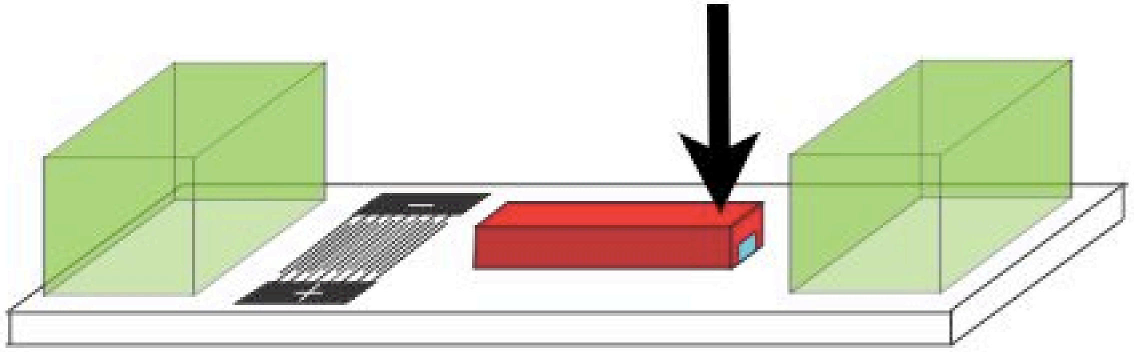


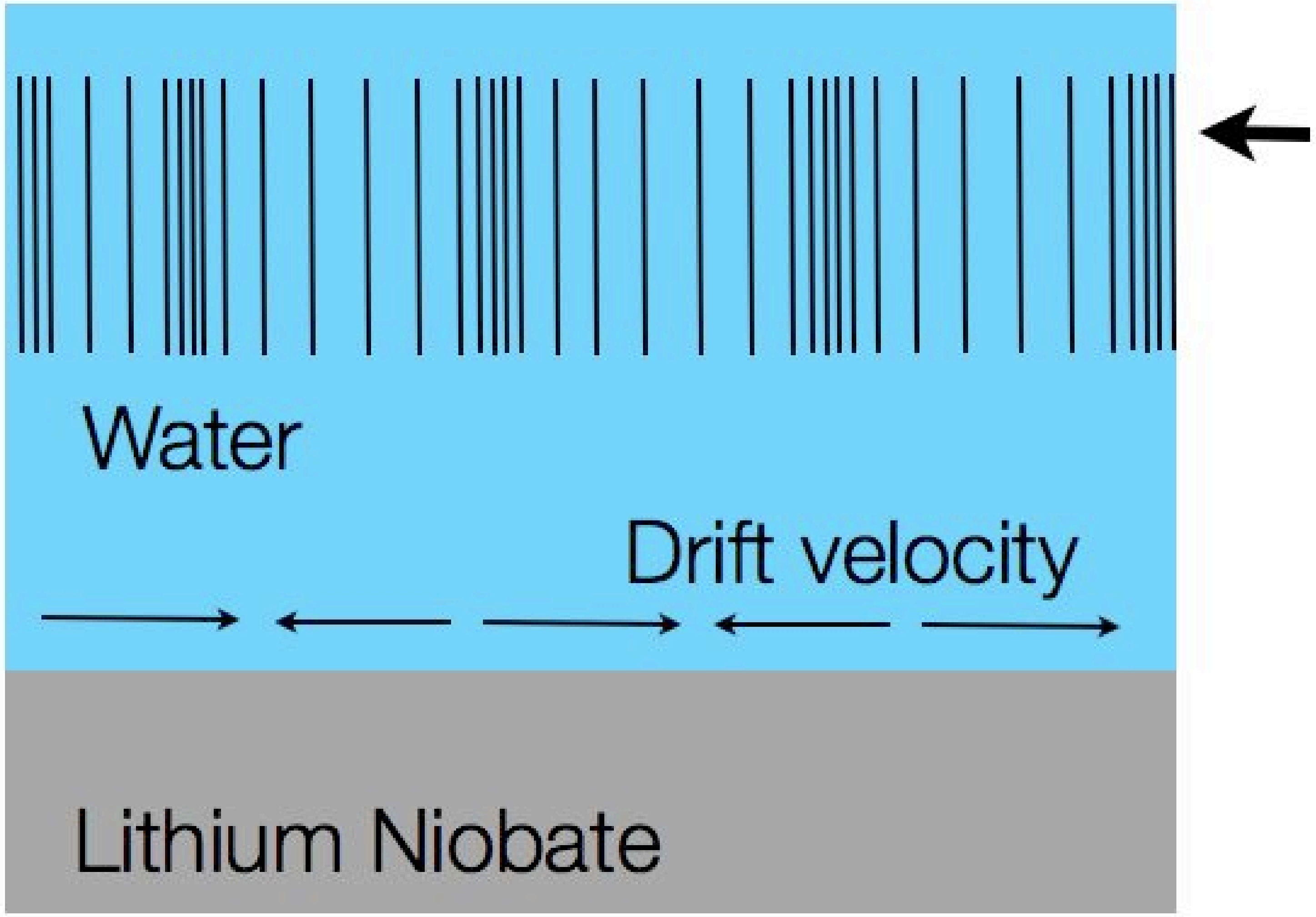
Bulk wave length in the liquid

$$\lambda_l = 47\mu m$$

Drift flow

Stationary bulk wave in a channel



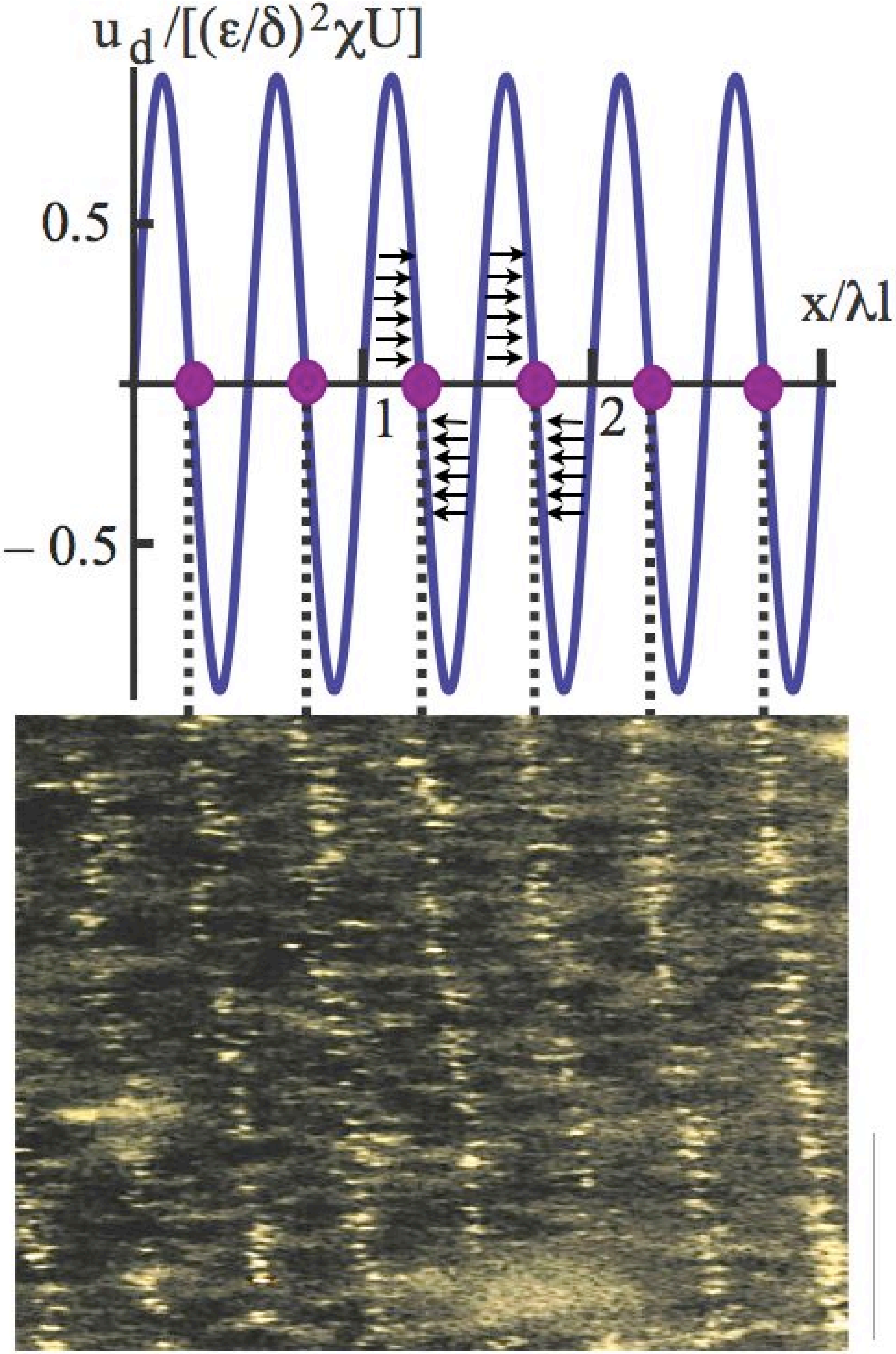


Stationary bulk wave in water

$$u_x|_{\beta y \to \infty} = 2\chi U \mathrm{e}^{i\omega t} \cos\left(\delta k x\right)$$

Drift flow

Theory – stationary wave

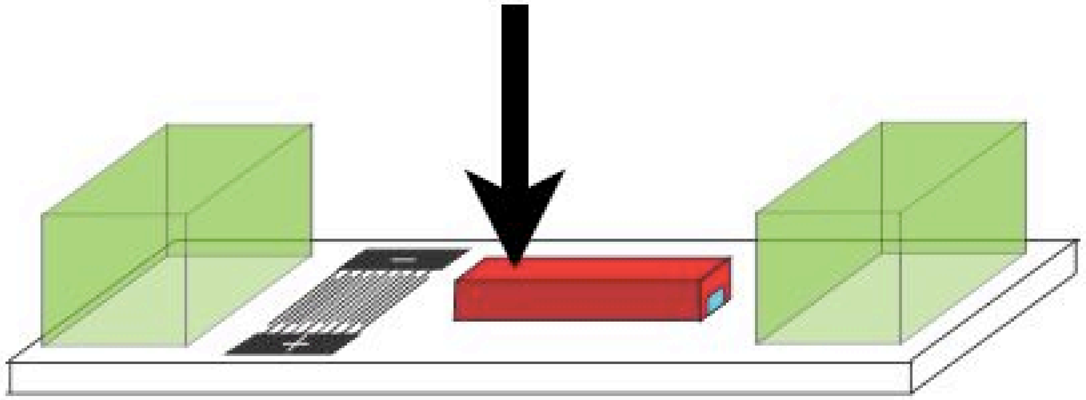


Drift velocity
($y \to \infty$)

$\lambda_l = 47\mu m$

● Stable stagnant point

Theory vs. experiment – stationary wave



SAW induced Stationary bulk wave

+

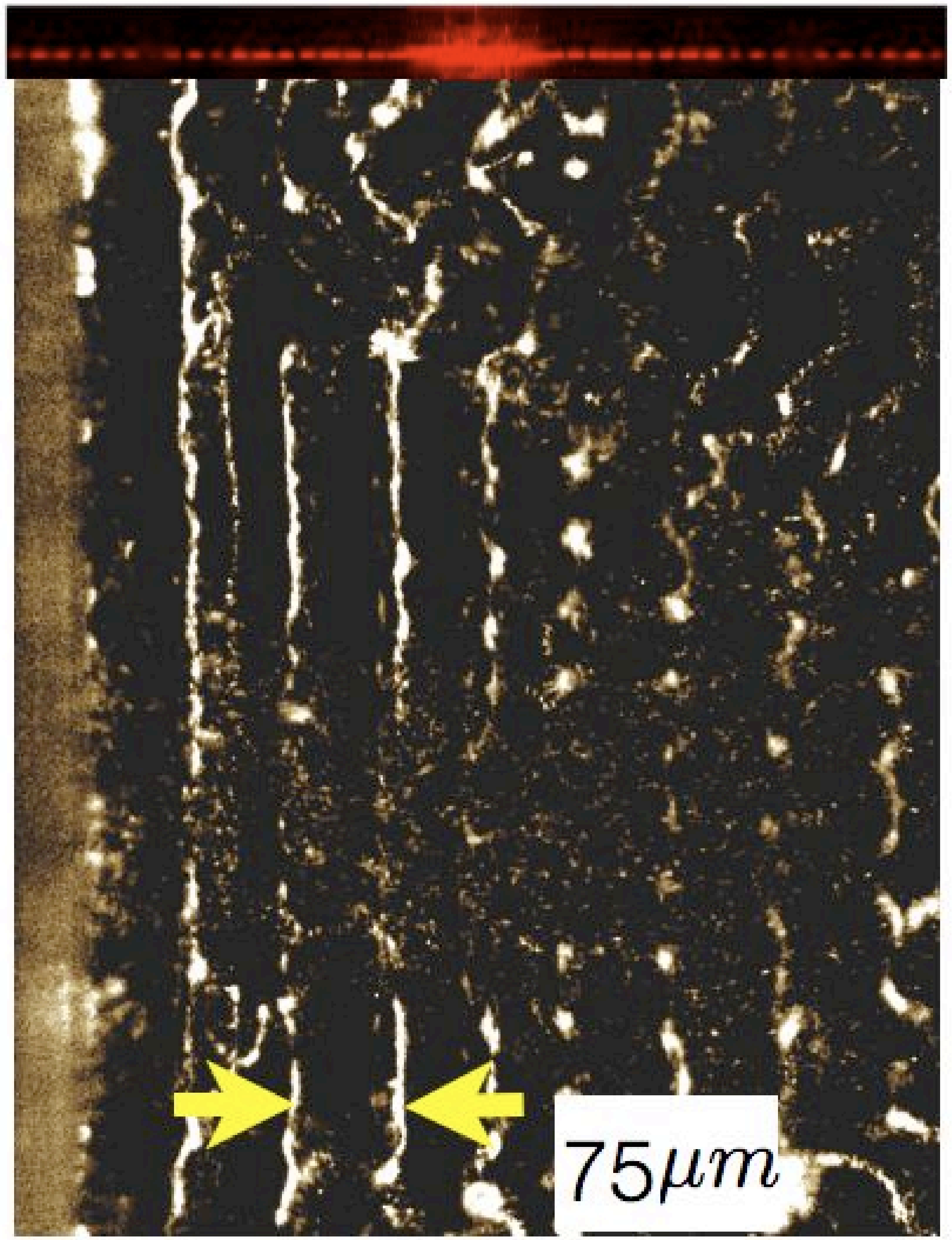
SAW

+

1 micron particles near the solid

↓

aggregate pattern



Bulk wave in water

$$\lambda_l = 47\mu m$$

wave length of SAW

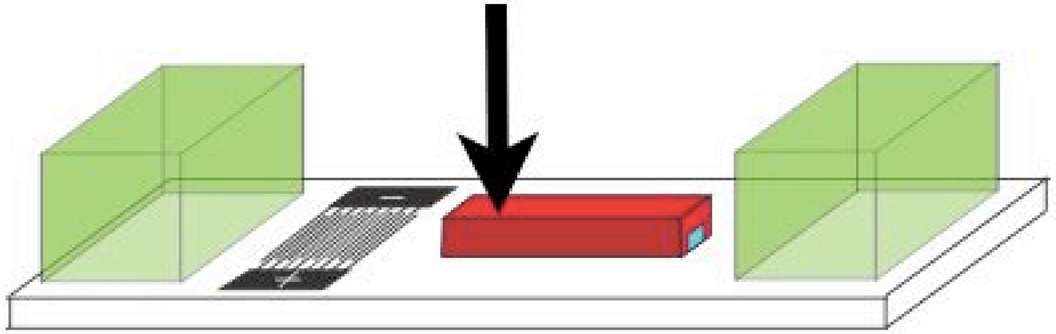
$$\lambda_{SAW} = 144\mu m$$

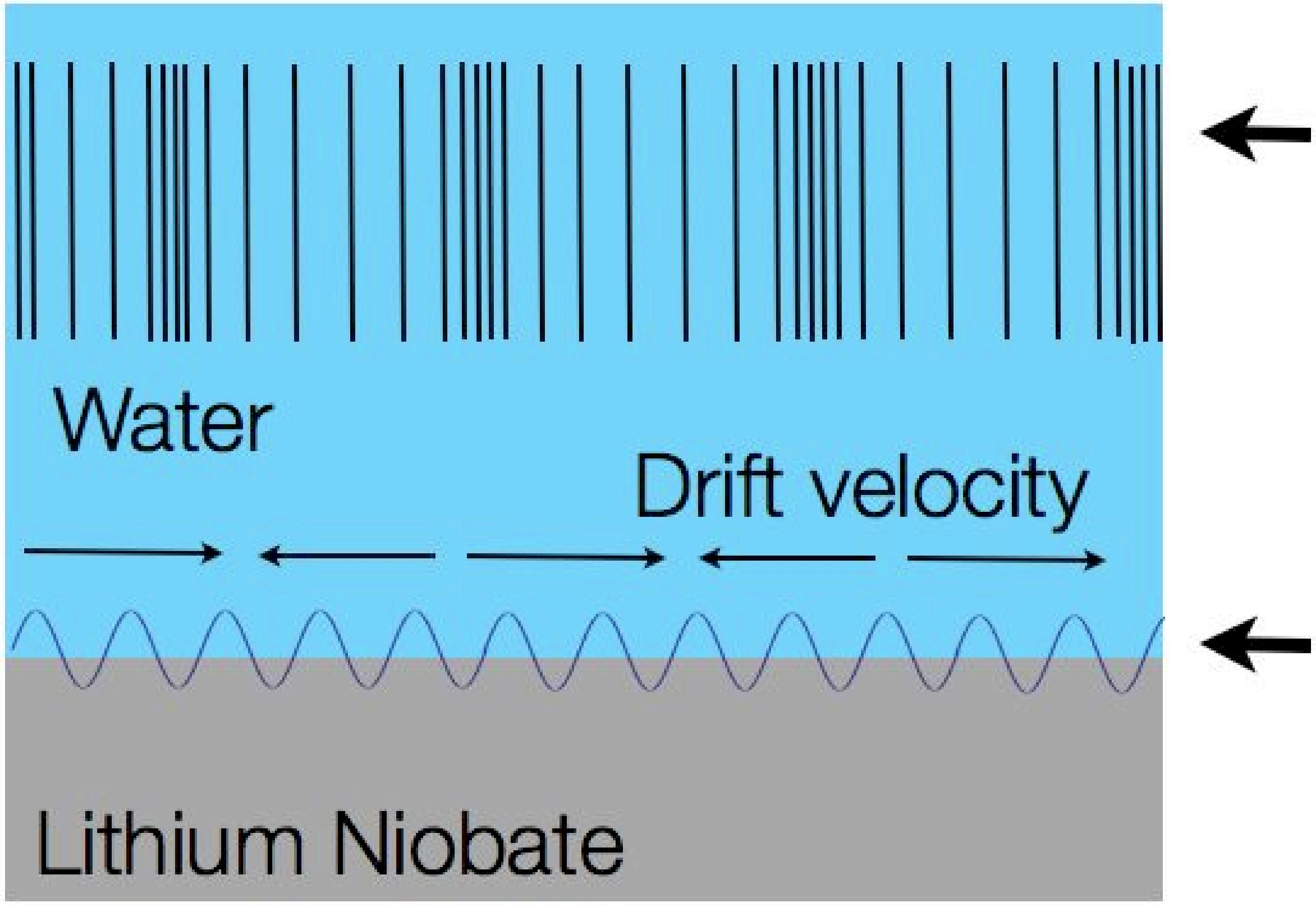
Drift flow

SAW + stationary bulk wave in a channel

Drift flow

SAW + stationary bulk wave in a channel





Standing bulk wave

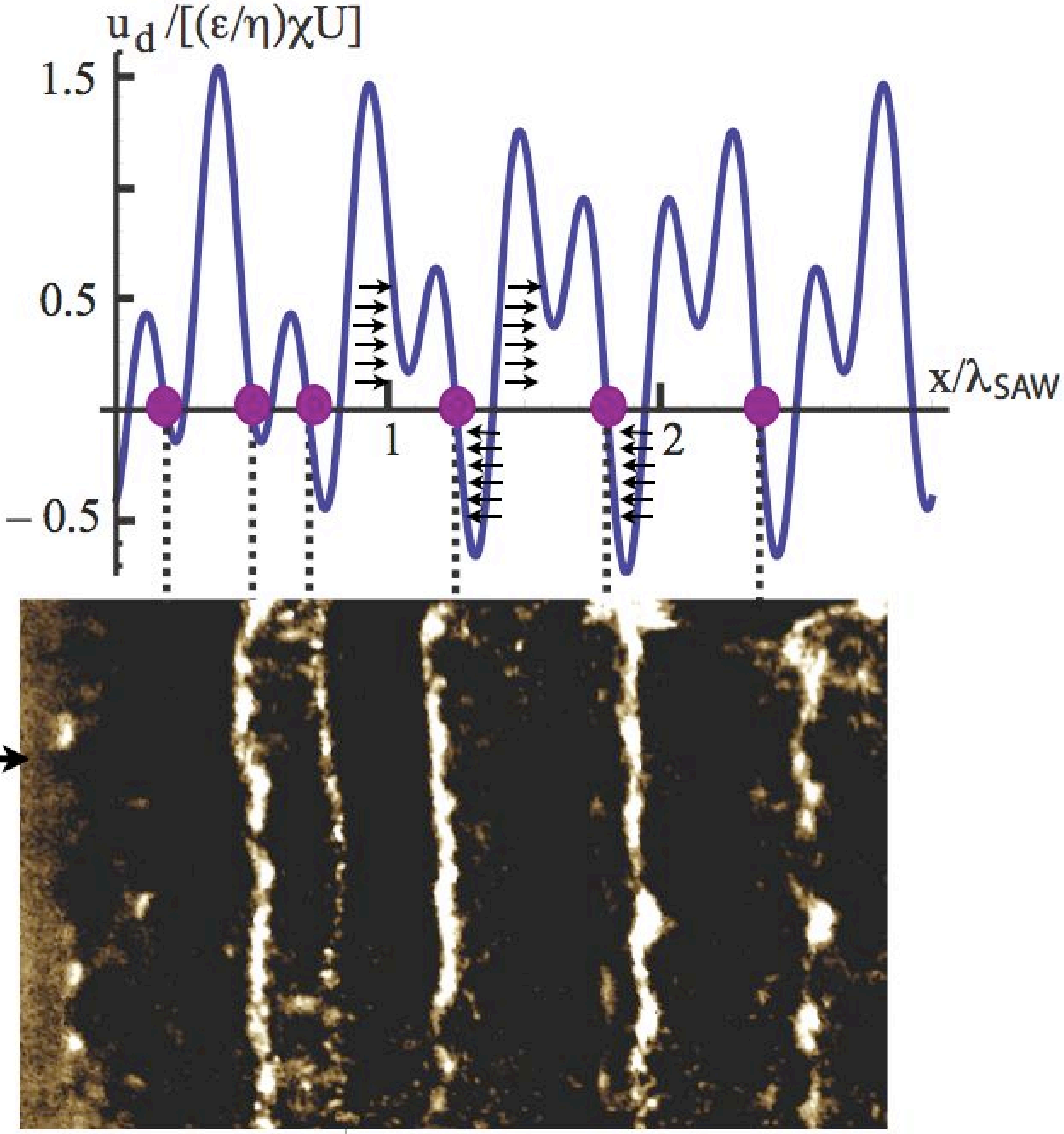
$$u_x|_{\beta y\to\infty} = 2\chi U \mathrm{e}^{i\omega t}\cos\left(\delta k x\right)$$

SAW

$$\begin{pmatrix} u_x \\ u_y \end{pmatrix}_{y=0} = \begin{pmatrix} \chi \mathrm{e}^{i(t-x)-\alpha x} \\ \eta^{-1}\mathrm{e}^{i(t-x+\varphi)-\alpha x} \end{pmatrix}$$

Drift velocity
$(y \to \infty)$

● Stable stagnant point



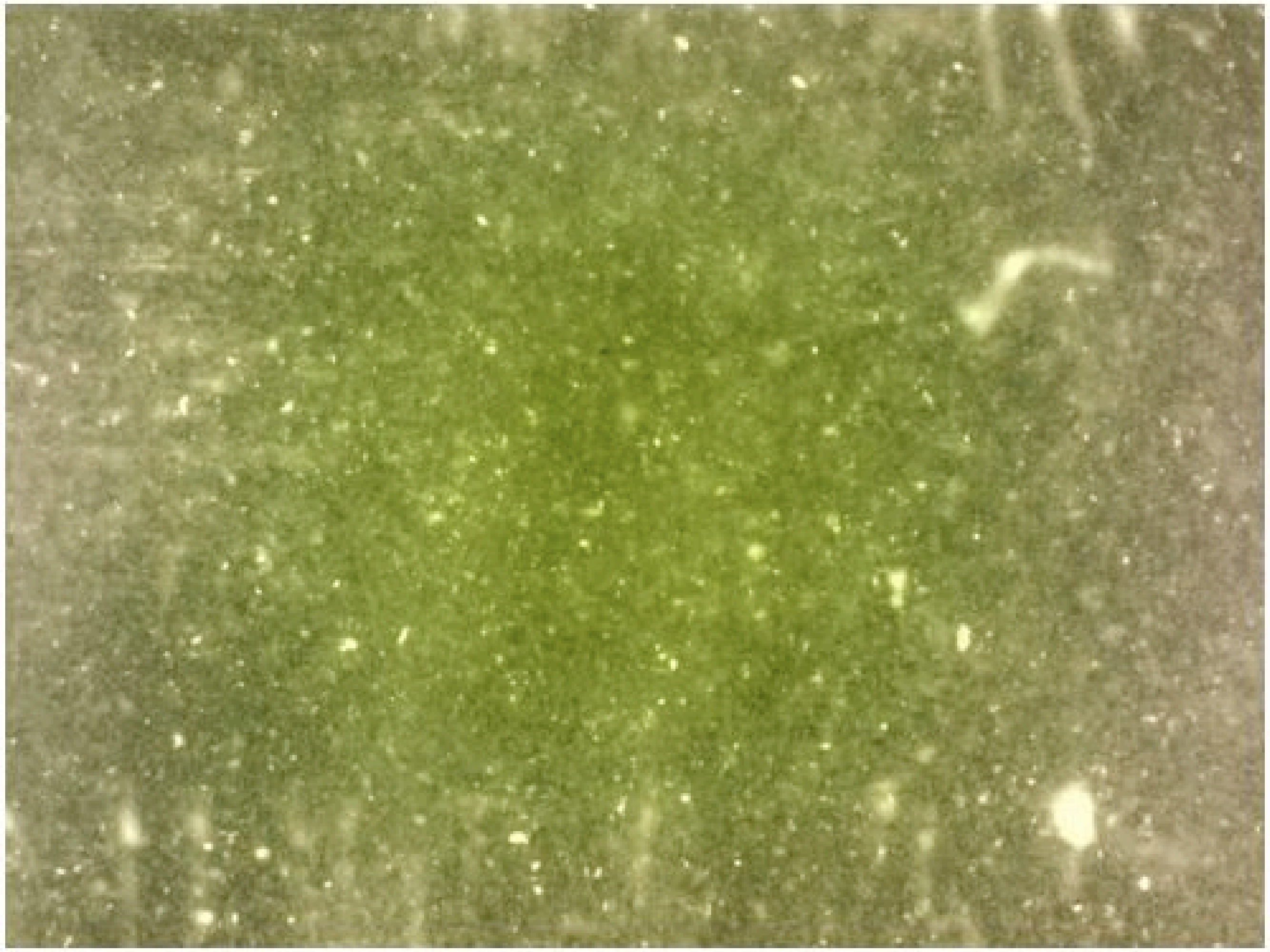
Channel edge

Theory vs. experiment –
SAW + stationary bulk wave

$\lambda_{SAW} = 144\mu m$

- **SAW excitation**

- Stationary wave generation (close structures)



*Manor, Friend and Yeo;
J. Fluid Mech.; submitted 2011

2nd order
Drift velocity

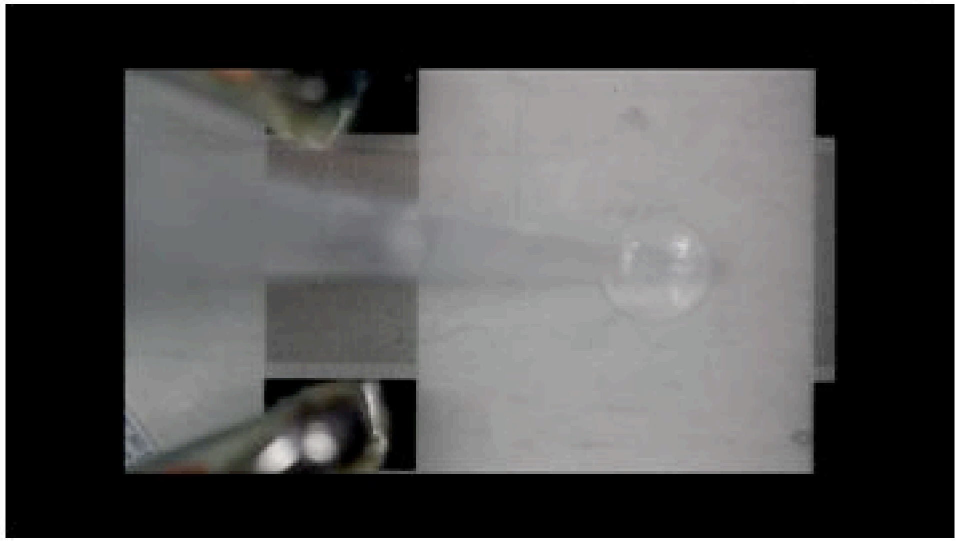
- Slip boundary condition

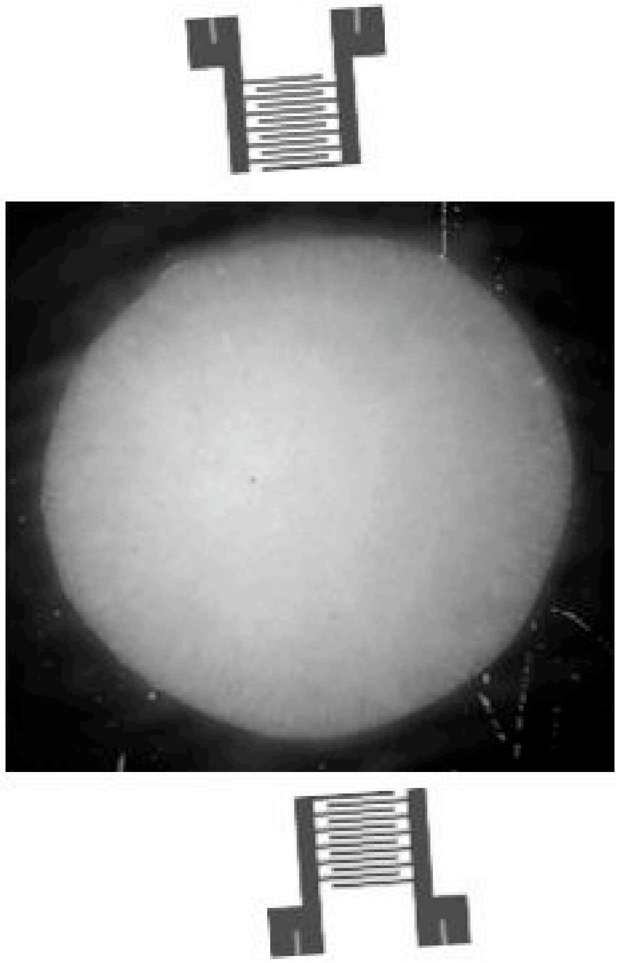
- aggregate patterning

Spreading films

Aggregate patterning

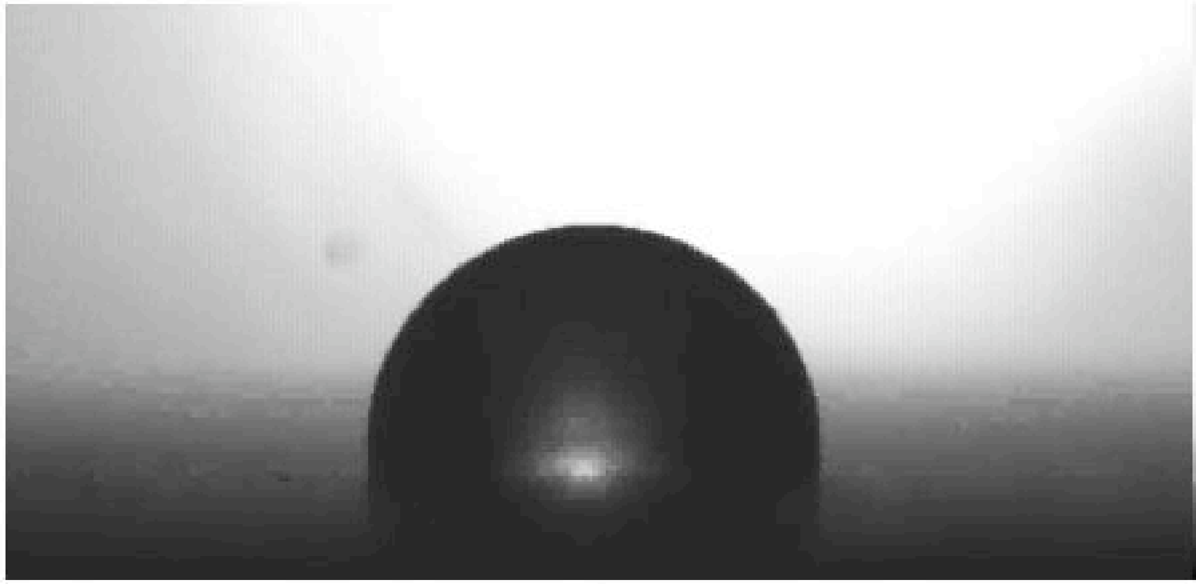
A. Rezk (MNRL)



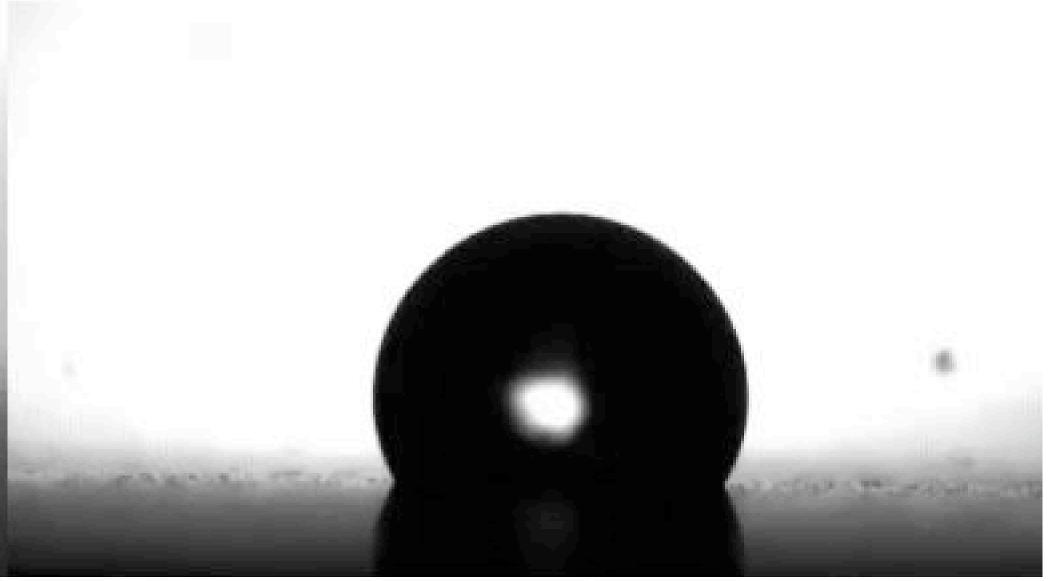


Applications:

- Drop microfluidics
- Micro electronics
- Particle separation
- Particle concentration

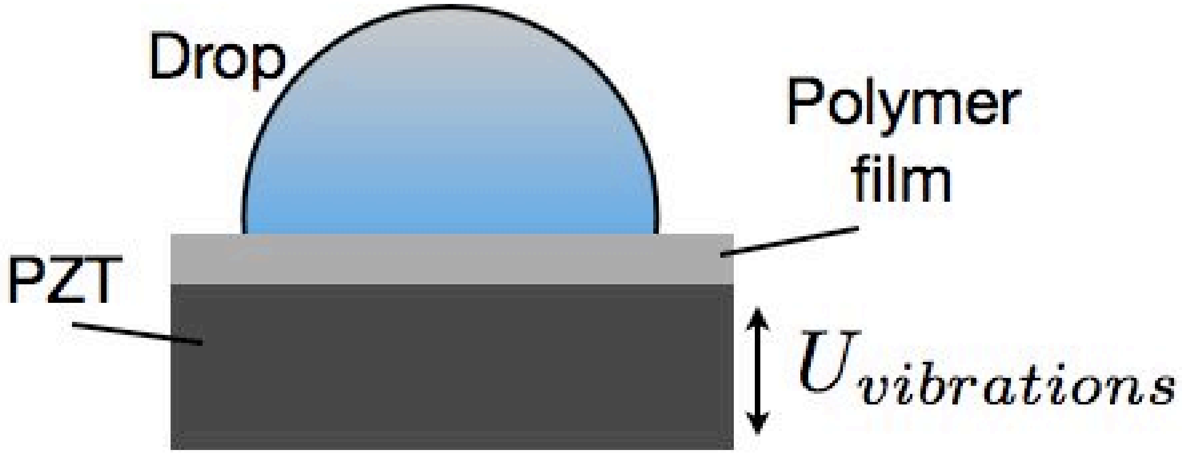


Hydrophilic substrate



Hydrophobic substrate

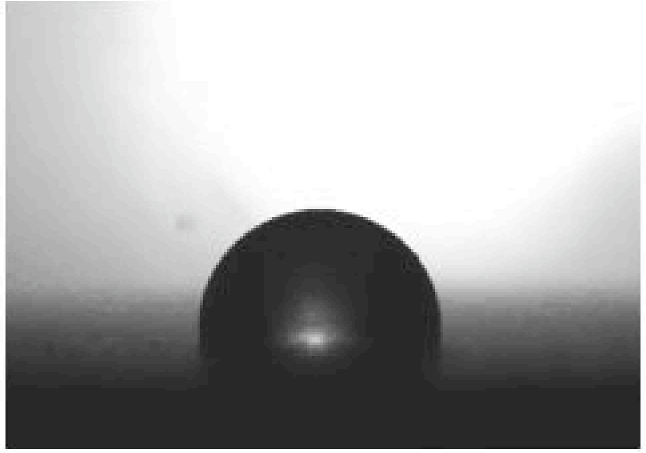
↔

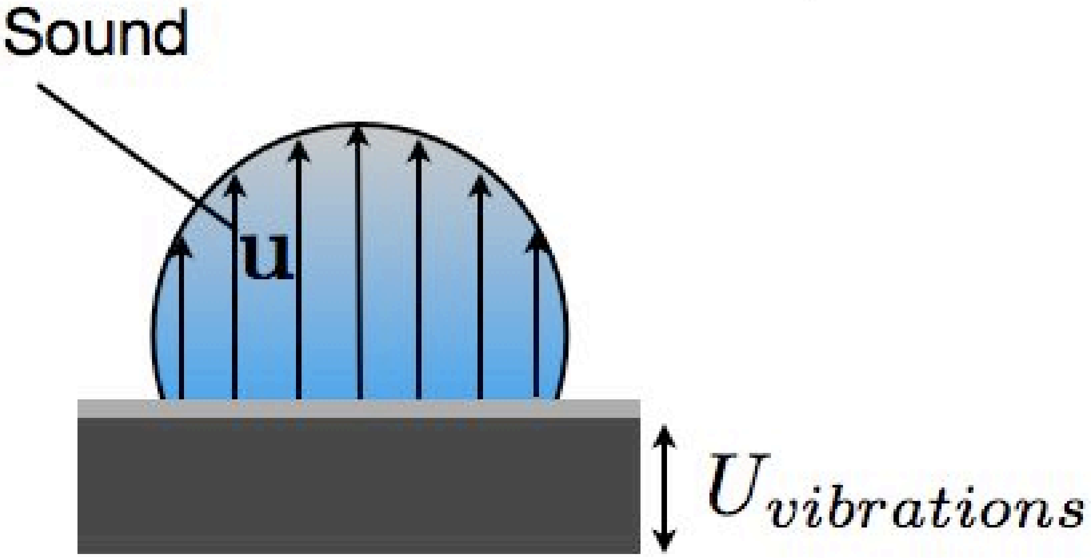


Spreading contact line

Spreading drops
(thickness mode waves)

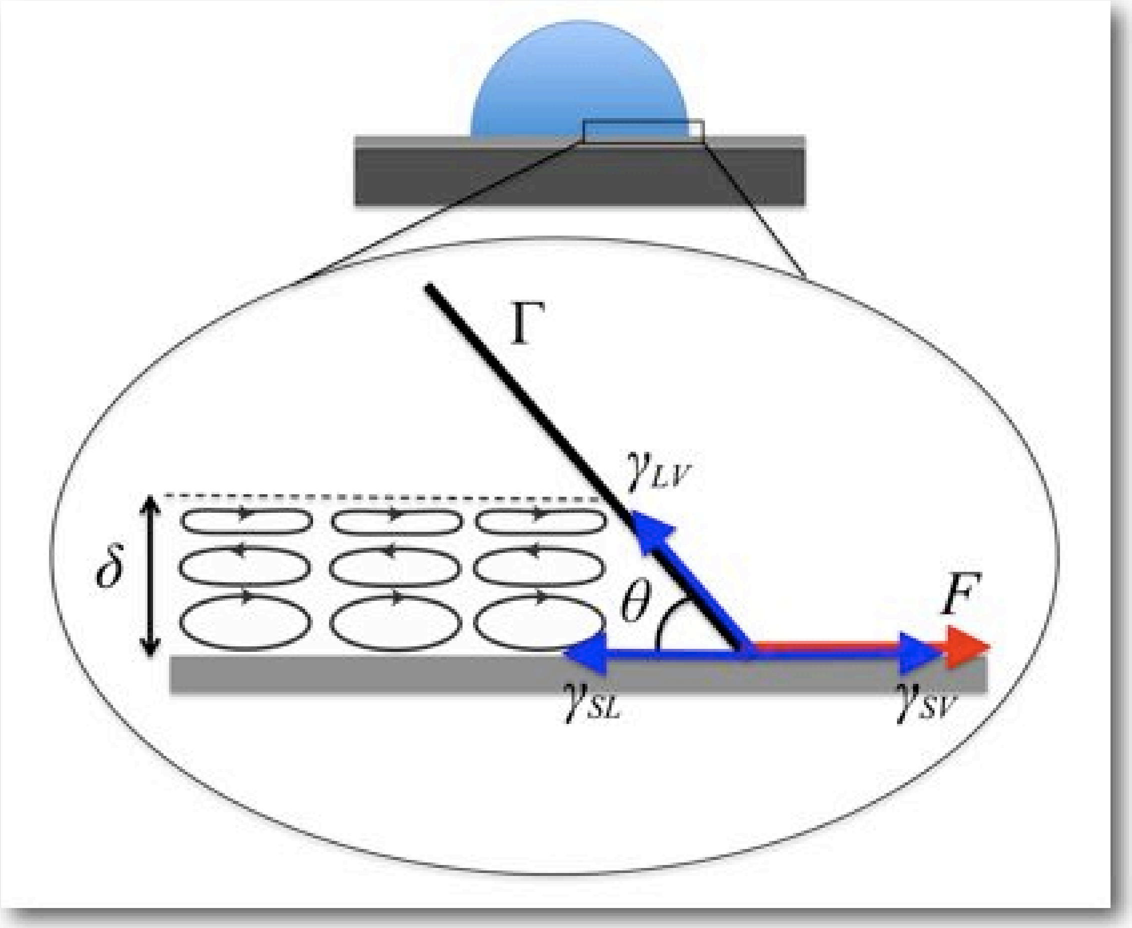
**Manor, Dentry, Friend and Yeo;
Soft Matter 2011, **7**, 7976-7979





$$F_{radiation} \sim \int_A \rho \mathbf{u}\mathbf{u} \cdot \mathbf{n} dx^2$$

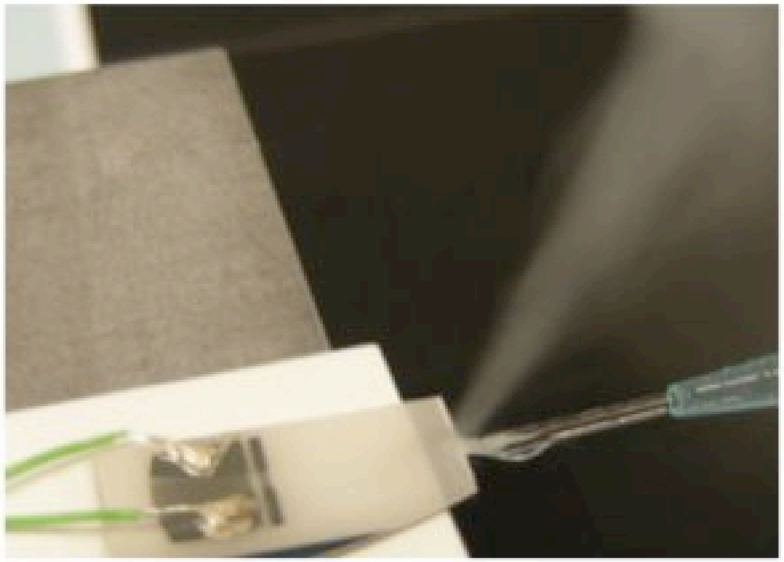
$$F_{streaming} \sim \int_V \rho \frac{\partial}{\partial \mathbf{x}} \cdot \mathbf{u}\mathbf{u} dx^3$$

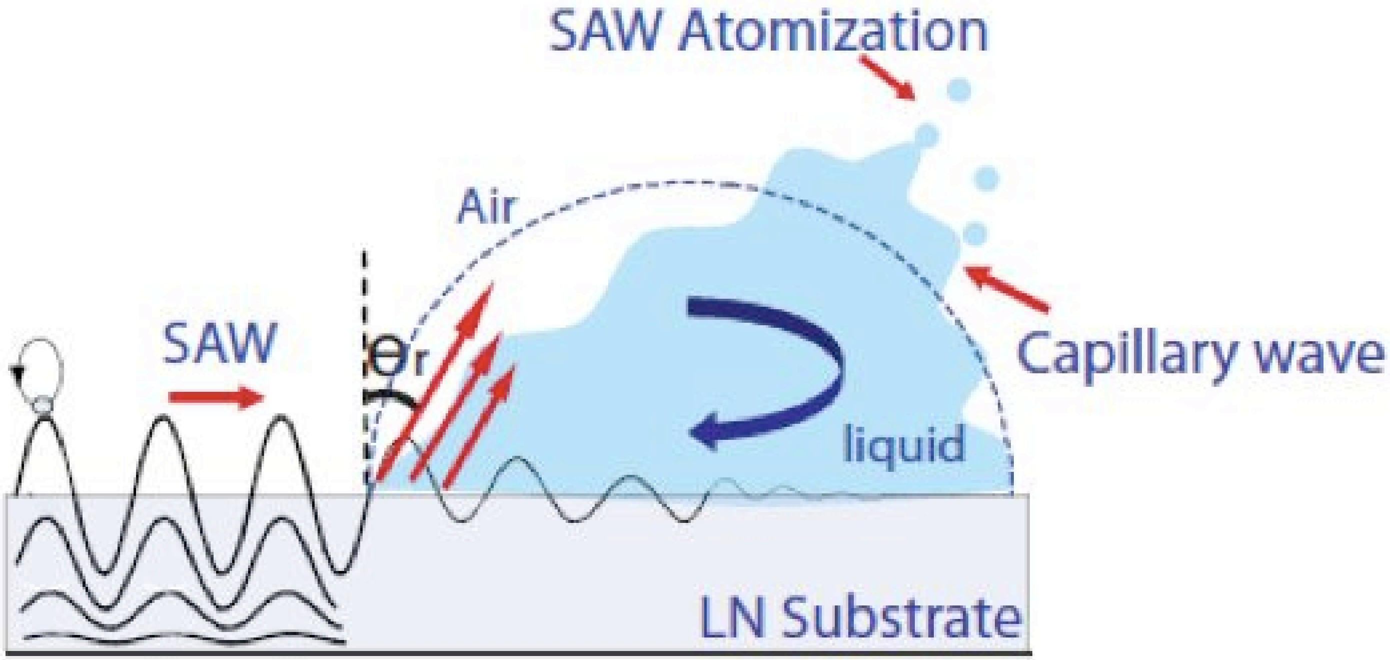


$$F \sim \rho\sqrt{2\mu/\rho\omega}U^2_{vibrations}R^2_{drop}cos^2\theta$$

Spreading contact line

Spreading drops
(thickness mode waves)



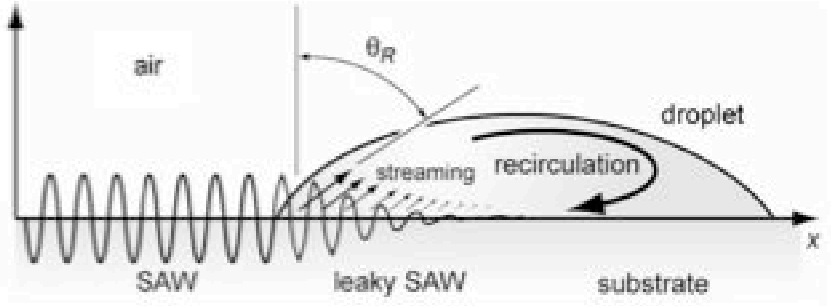


Surface acoustic wave →

U~1 m/s, Water

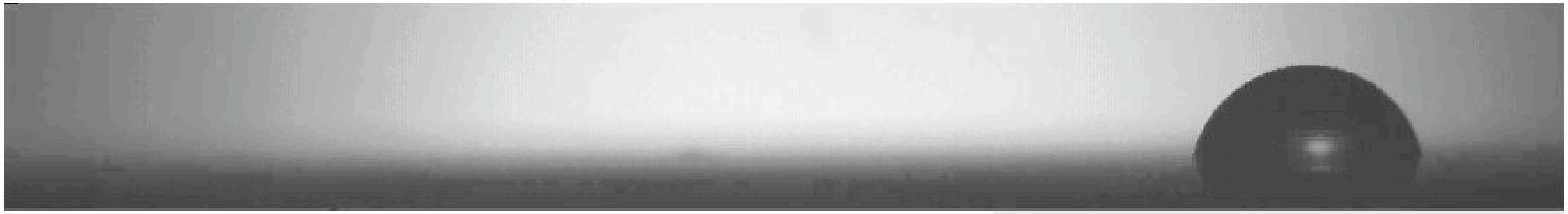
D. Collins (MNRL)

Spreading contact line | Spreading atomization film (SAW)



← Surface acoustic wave

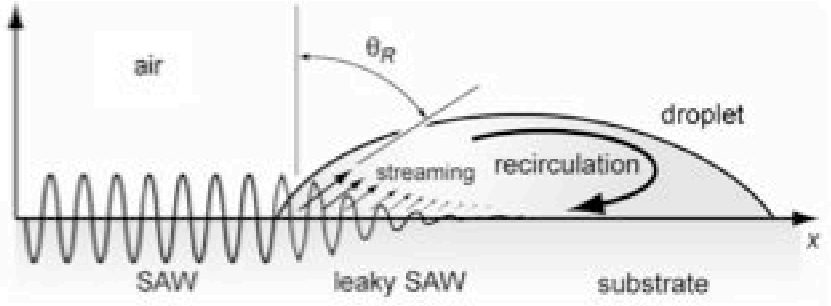
U=16 cm/s, Water



U=16 cm/s, Glycerol

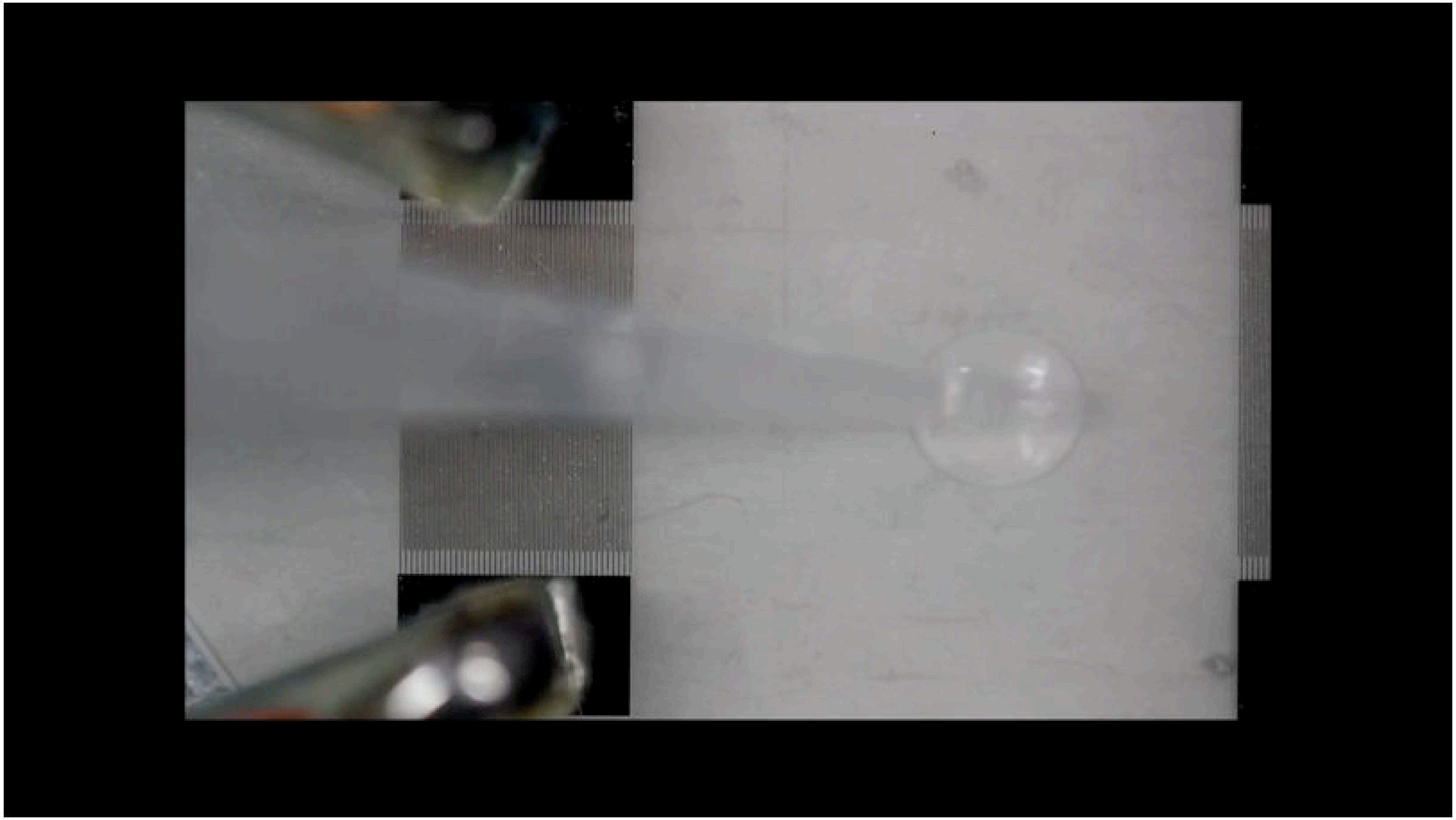
Spreading contact line

Translating drops (SAW)



U~10-0.1 mm/s,
Silicon oil

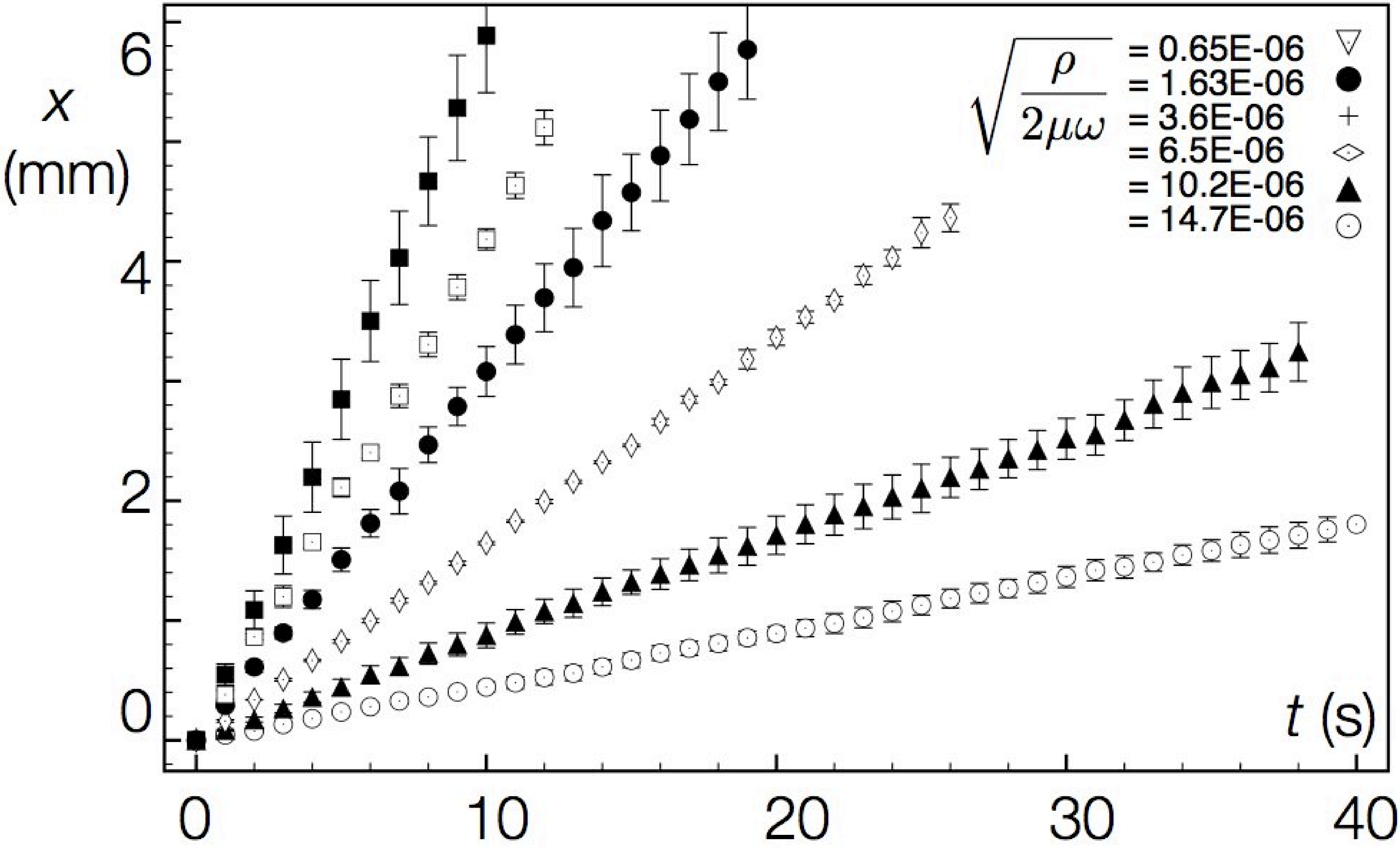
Surface
acoustic
wave
→



A. Rezk (MNRL)

Spreading contact line

Spreading film vs. moving drop (SAW)

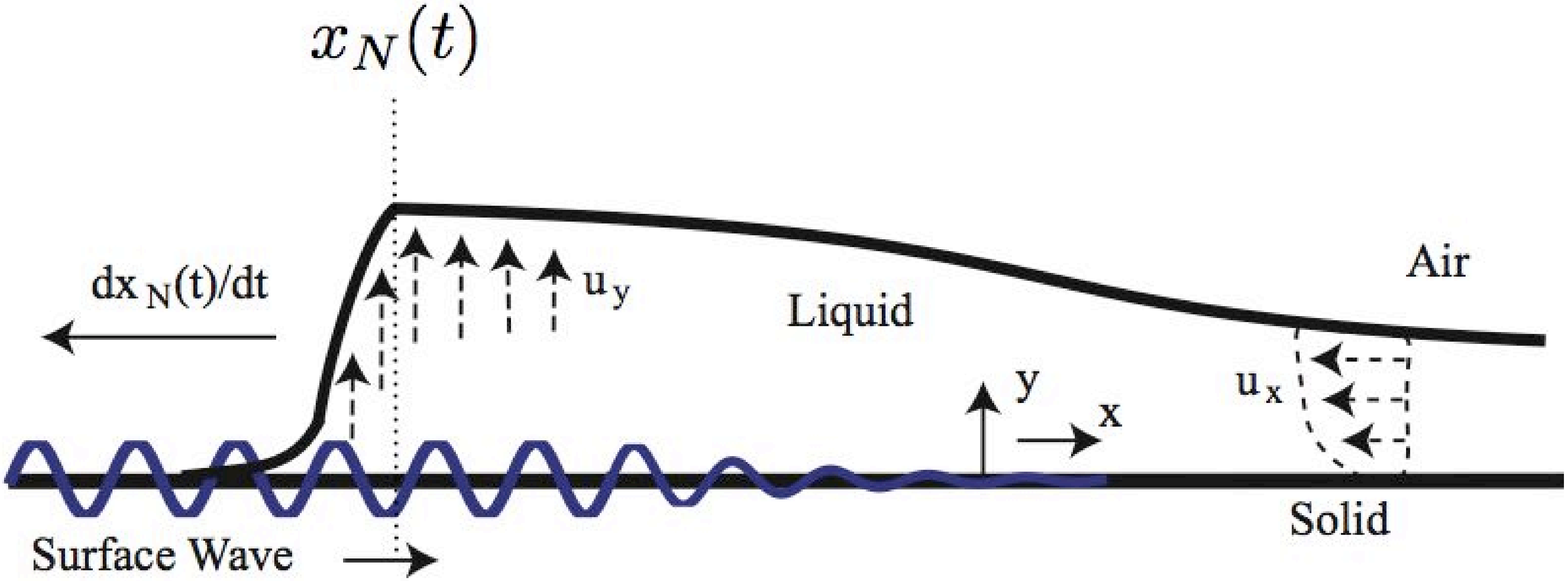


Spreading contact line

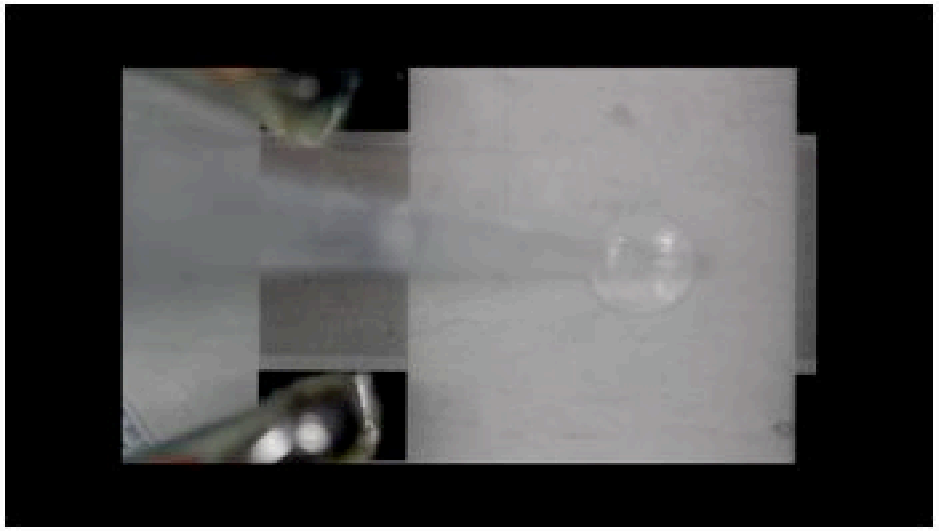
Theory vs. Experiment –
film spreading displacement

Film spreading velocity

$$\frac{dx_N}{dt} \approx -\frac{\chi}{6\log 2}\sqrt{\frac{\rho}{2\mu\omega}}[cos(\varphi) - sin(\varphi)]U^2$$



Spreading contact line



Dimensionless quantities

$\eta \equiv \beta^{-1}/k_{SAW}^{-1} \ll 1$ (boundary layer thickness) / (SAW wave length)

$\epsilon \equiv U/C_{SAW} \ll 1$ Mach number: (Flow velocity) / (Sound speed)

Scaling

$$\left.\begin{array}{l} t \to t/\omega,\ x \to k^{-1}x,\ y \to \beta^{-1}y,\ h \to \beta^{-1}h,\ u_x \to Uu_x,\ u_y \to \eta U u_y, \\ \\ \alpha \to k\alpha,\ \psi, \psi \to \beta^{-1}U,\ Q \to \beta^{-1}U, p \to (\mu\beta^{-1}U/\eta)p,\ \beta^{-1} \equiv \sqrt{2\mu/\rho\omega} \end{array}\right\}$$

SAW boundary condition

$$\begin{pmatrix} u_x \\ u_y \end{pmatrix}_{y=0} = \begin{pmatrix} \chi \mathrm{e}^{i(t-x)-\alpha x} \\ \eta^{-1}\mathrm{e}^{i(t-x+\varphi)-\alpha x} \end{pmatrix}$$

B.C.s at the gas/liquid interface (neglecting surface tension)

$$\begin{pmatrix} \partial_y u_x \\ \partial_y u_y \end{pmatrix}_{y=h} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}, \qquad \beta^{-1} \ll h \ll \lambda$$

Expansion

$$u_x = \sum_n f_n u_{x,n}, \qquad u_y = \sum_n f_n u_{y,n}, \qquad \psi = \sum_n f_n \psi_n,$$

$$\epsilon \ll \epsilon/\eta \ll 1 \ll 1/\eta$$

For water/oil at *O*(1-100MHz) SAW

Stream function components

$O(1/\eta)$: $\frac{1}{2}\partial_y^4\psi_{-1} = \partial_t\left(\partial_y^2\psi_{-1}\right)$ .... **Unidirectional flow**

$O(1)$: $\frac{1}{2}\partial_y^4\psi_0 = \partial_t\left(\partial_y^2\psi_0\right)$ .... **Classic Schlichting leading order flow**

$O(\epsilon/\eta)$: $\frac{1}{2}\partial_y^4\left\langle\psi_1\right\rangle = -\left\langle\partial_x\overline{\psi}_{-1}\partial_y^3\overline{\psi}_0\right\rangle$ .... **drift flow**

$O(\epsilon)$ .... **Classic Schlichting drift flow**

$\langle\rangle$ Averaging over time

$\overline{\psi} \equiv Real(\psi)$

Formalism

Stream function and asymptotic analysis

Key results (dimensional) - flow induced by SAW

$$Q = Q_{SW} - \frac{h^3}{3}\partial_x p$$

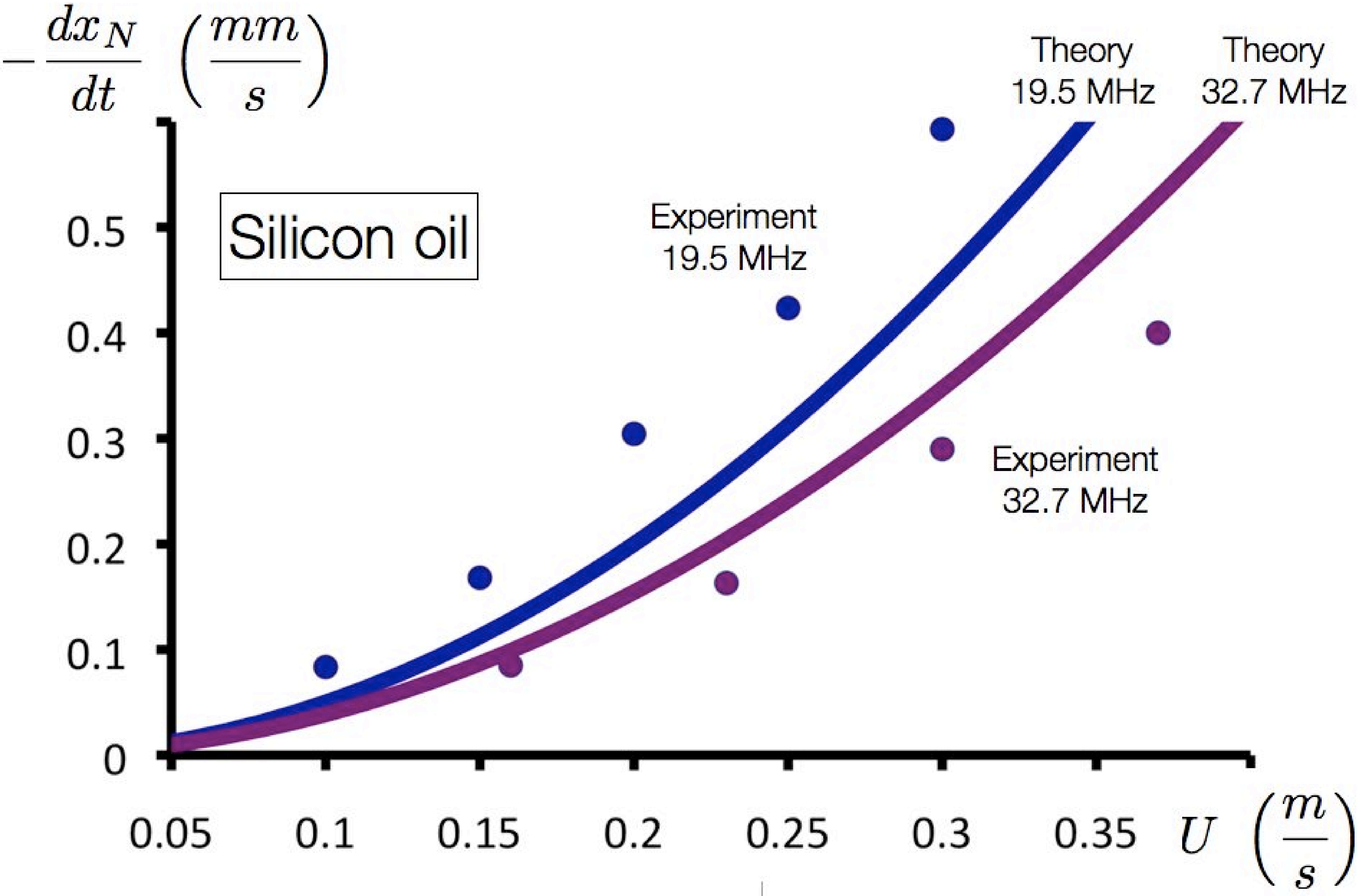
$$\langle u_y \rangle |_{y=h} \approx \frac{\epsilon}{\eta}\frac{\chi\alpha}{3} U \mathrm{e}^{-2\alpha(x - x_N(t))}[cos(\varphi) - sin(\varphi)]h$$

Thin Film flow (dimensional)

$$\frac{dh}{dt} = -\frac{d}{dx}Q \qquad \frac{dh}{dt} = \langle u_y \rangle |_{y=h}$$

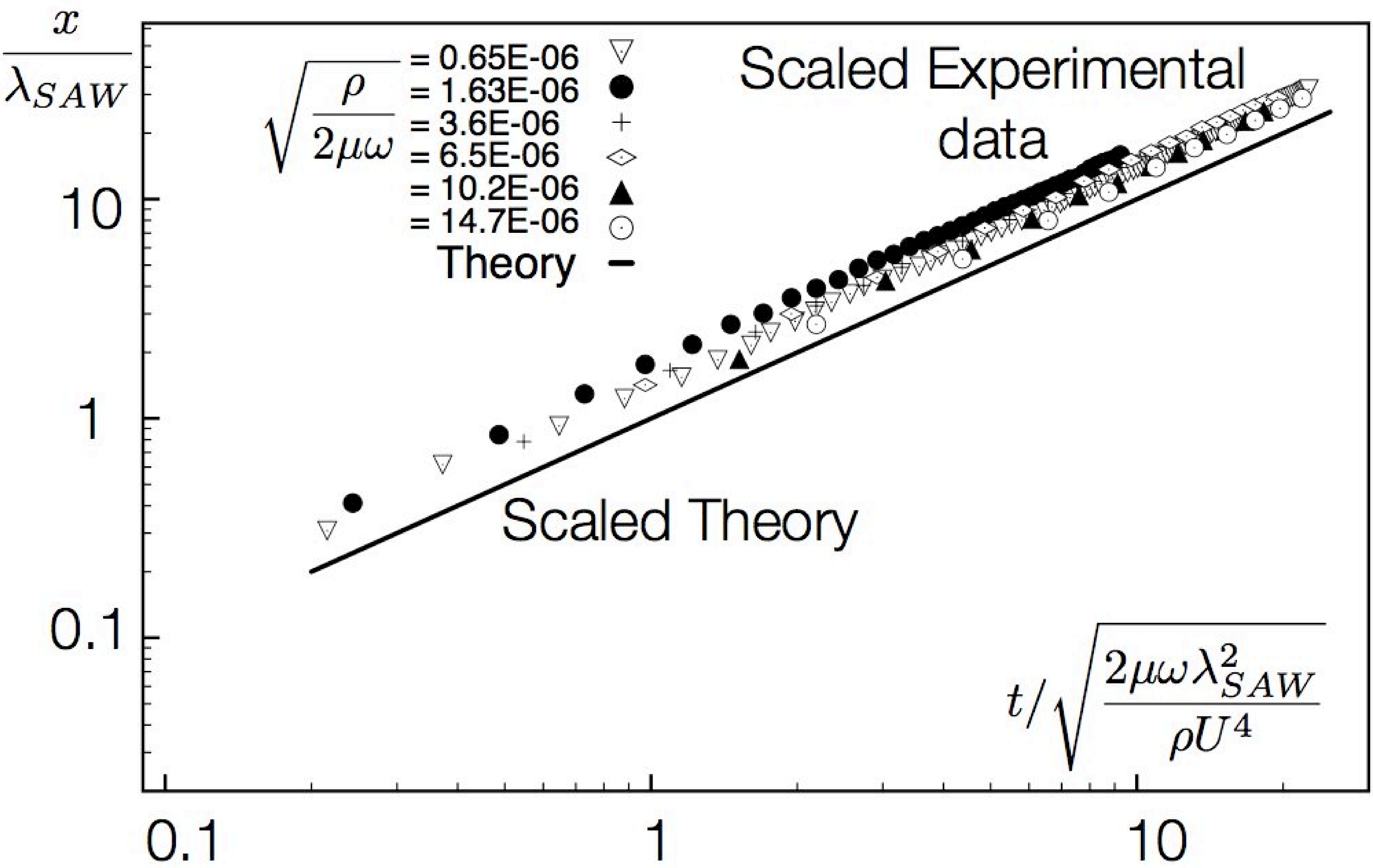
Film spreading velocity (dimensional)

$$\frac{dx_N}{dt} \approx -\frac{\chi}{6\log 2}\sqrt{\frac{\rho}{2\mu\omega}}[cos(\varphi) - sin(\varphi)]U^2$$



Spreading contact line

Theory vs. Experiment – film spreading velocity



Spreading contact line

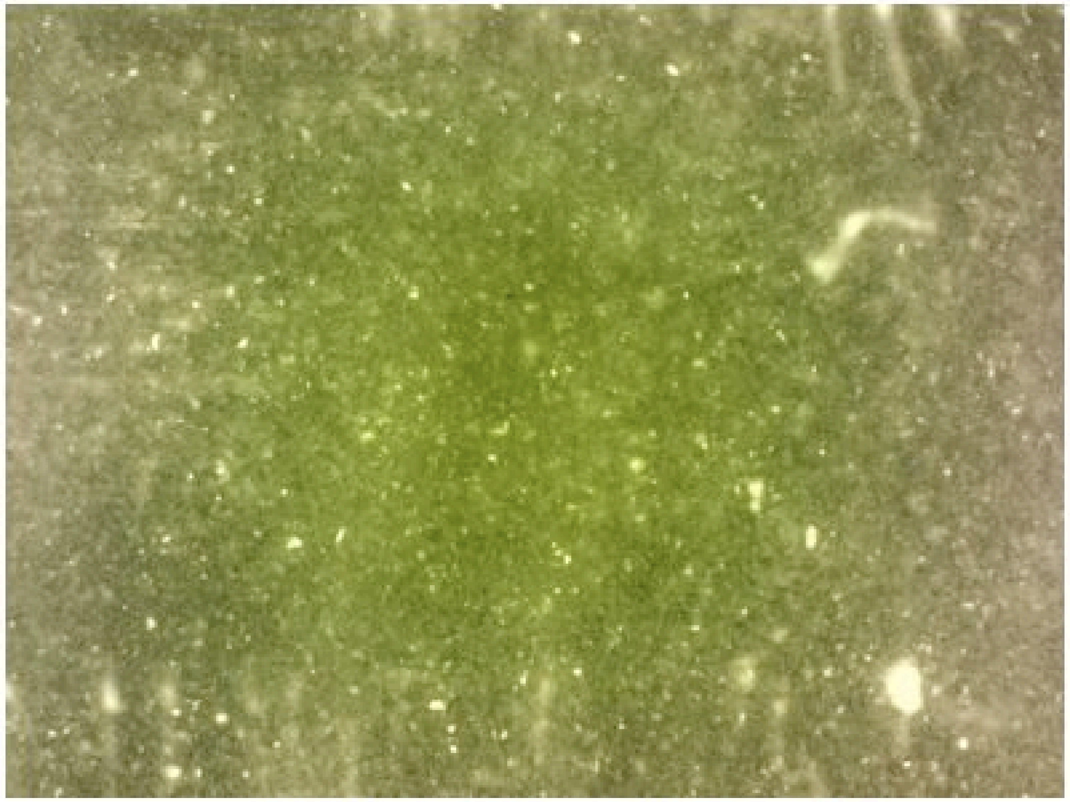
Data collapse

High frequency SAW excitation

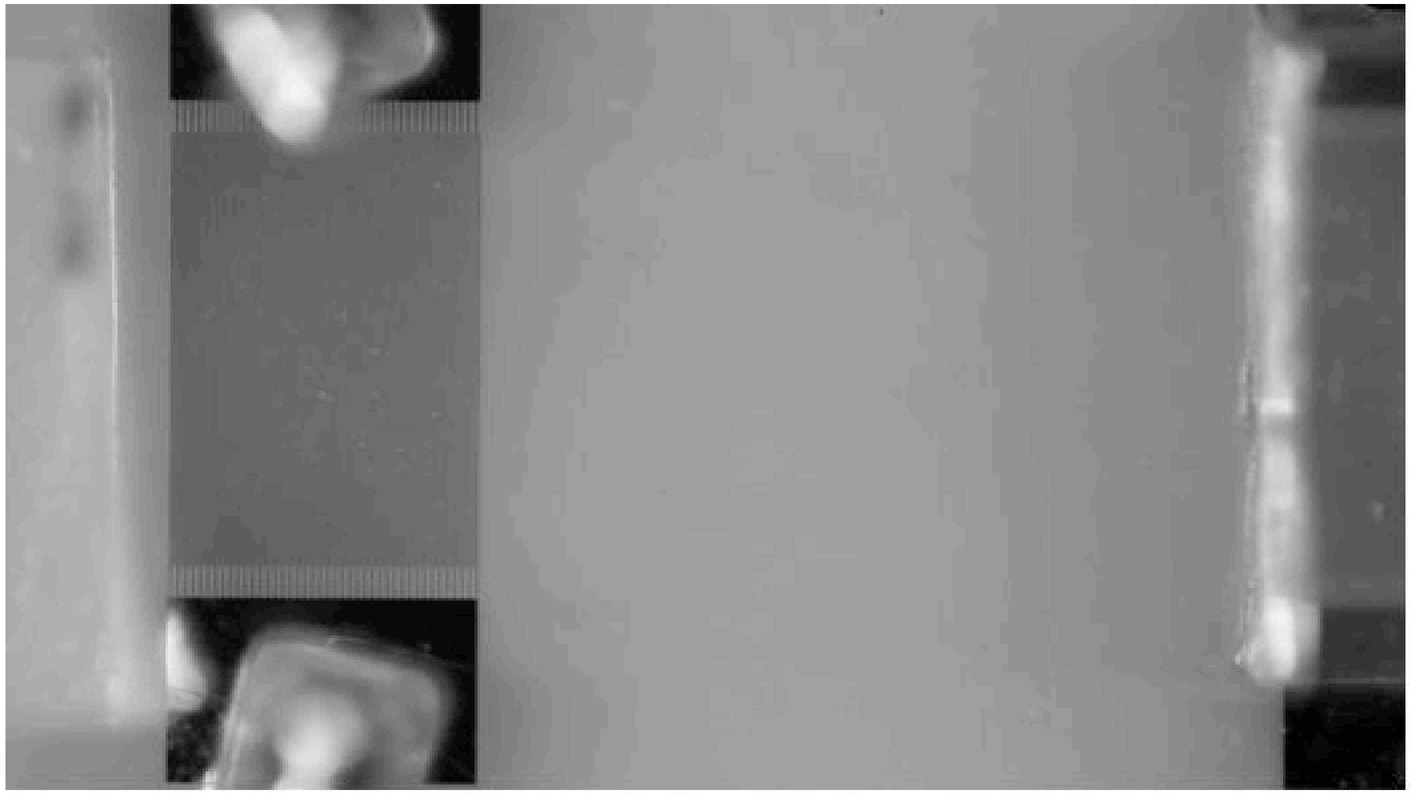
Drift Flow

- Slip boundary condition
- aggregate patterning

- Contact line displacement
- Spreading films, drops



*Manor, Friend and Yeo;
J. Fluid Mech.; submitted 2011



**Rezk, Manor, Friend and Yeo;
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